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Unconditional Mean, Volatility and the Fourier-GARCH Representation

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Abstract

This paper proposes a new model called Fourier-GARCH that is a modification of the popular GARCH(1,1). This modification allows for time-varying first and second moments via means of Flexible Fourier transforms. A nice feature of this model is its ability to capture both short and long run dynamics in the volatility of the data, requiring only that the proper frequencies of the Fourier transform be specified. Several simulations show the ability of the Fourier series to approximate breaks of an unknown form, irrespective of the time or location of breaks. The paper shows that the main cause of the long run memory effect seen in stock returns is the result of a time varying first moment. In addition, the study suggests that allowing only the second moment to vary over time is not sufficient to capture the high persistence observed in lagged returns.

Keywords:

ARCH/GARCH, Structural change, Unconditional volatility.

JEL classification:

G12, G29.

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1. Introduction

Recently there has been an upsurge interest in modeling the nonstationarities present in the volatility of financial data. The clustering and the persistence of volatility of asset returns have been well documented. The IGARCH model of Engle and Bollerslev (1986) for instance, describes in a parsimonious way the high persistence in the conditional volatility of stock returns while the underlying process remains strictly stationary. Alternatively, Granger (1980) and Granger and Joyeux (1980) model the long memory or the long range dependence of a series of log-returns as a fractionally integrated process to allow the autocorrelation functions to decay very slowly, in a fashion characteristic of stock returns. However, seminal papers from Granger and Joyeux (1986) and Lamoureux and Lastrapes (1990) and more recently from Diebold and Inoue (2001), Mikosch and Starica (2004), Starica and Granger (2005), and Perron and Qu (2007) argue that the high persistence close to unit root and long memory both in the first and the second moments may actually be caused by structural changes in the level or slope of an otherwise locally stationary process of the long-run volatility. Diebold and Inoue (2001) argue that this is due to switching regimes in the data. Mikosch and Starica (2004) provide theoretical evidence that changes in the unconditional mean or variance induce the statistical tools (e.g., sample ACF, periodogram) to behave the same way they would if used on stationary long-range dependent sequences. Starica and Granger (2005) also deliver evidence against global stationarity. Finally, Perron and Qu (2007) conclude that the S&P 500 return series is best described as a stationary short memory process contaminated by mean shifts.

These results imply that a good model for volatility should take into account the possibility of a time varying unconditional second-moment and possibly, of a time varying first moment as well.

Engle and Rangle (2008) propose the Spline-GARCH to model long-run volatility nonparametrically using an exponential quadratic spline. However, they do so only for the second moment. Further, Starica and Granger (2005) use step functions to approximate nonstationary data locally by stationary models. They apply their methodology to the S&P 500 series of returns covering a period of seventy years of market activity and find that most of the dynamics are concentrated in shifts of the unconditional variance.

However, these models pose several problems. While spline functions may lead to over fitting, step functions may not give smooth approximations. Even major breaks, such as the stock market crash of 1929 and the oil price shocks of the 1970s did not display their full impact immediately. Structural changes may take longer to extinguish which suggests they need to be modeled as smooth or gradual changing processes. These arguments motivate the present study to propose a new approach to model the long-run first and second moments as smooth processes. The paper denotes the new process Fourier-GARCH because it uses the Flexible Fourier transform of Gallant (1981) (i.e., an expansion of a periodic function in terms of an infinite sum of sines and cosines). The basic model can be extended to incorporate the long-run volatility in the mean model. Flexible Fourier transforms have been used in the literature to approximate nonlinear structures in several ways. For instance, Becker et al. (2001) use Fourier transforms to model inflation and money demand as having smooth changes in the intercept. Also, Enders and Lee (2006) and Becker et al. (2006) propose new unit root and stationarity tests that use the Fourier approximation to model the unknown shape of the structural breaks in macro time series. The main advantage is that the issue of estimating the shape and location of the breaks reduces to selecting the proper frequency of the Fourier sine and cosine terms. A section below details how Fourier transforms can be used to approximate various types of breaks.

The study applies the new model to several of the largest stocks from S&P 500 to estimate volatility persistence in stock returns. Based on the discussion above, the paper considers several competing models. The basic Fourier-GARCH model specifies a constant first-moment, while the second-moment changes smoothly over time. A first extension to the basic model allows both the first and the second moments to vary over time, while a second extension incorporates the long run volatility in the model for the mean. The paper checks for each model the sum of the estimated coefficients in the equation for conditional volatility to assess the so called long-memory effect. The results show that allowing only the second moment to vary over time does not significantly reduce the persistence effect. In fact, the difference between this model and the simple GARCH(1,1) is negligible. However, the extended model that allows the first moment to vary over time as well, reduces the persistence effect by more than half of the value suggested by GARCH(1,1). The evidence suggests that the persistence effect seen in stock returns is mainly a result of the misspecification of the model for the mean.

The paper is structured as follows. Section 2 discusses in more detail the performance of the Fourier series to approximate various types of structural breaks. Section 3 introduces the basic Fourier-GARCH model and its extensions. Section 4 discusses the empirical estimates of the long memory effect using four different models and section 5 concludes.

2. Nonlinear Trend Approximation with Fourier Transforms

The general approach to account for breaks is to approximate them using dummy variables. However, this approach has several undesirable consequences. First, one has to know the exact number and location of the breaks. These are not usually known and therefore need to be estimated. This in turn introduces an undesirable pre-selection bias (see Maddala and Kim, 1998). Second, use of dummies suggests sharp and sudden changes in the trend or level. However, for low frequency data it is more likely that structural changes take the form of large swings in the data which cannot be captured well using only dummies. Breaks should therefore be approximated as smooth processes (see Leybourne et al., 1998 and Kapetianos et al., 2003).

Flexible Fourier transforms, originally introduced by Gallant (1981), are able to capture the essential characteristics of one or more structural breaks using only a small number of low frequency components. This is true because a break tends to shift the spectral density function towards frequency zero. Below is illustrated the ability of Fourier transforms to capture nonlinear trends.

Using a simple form for the mean model, one can allow the intercept μ_t to be a deterministic function of time:

$$y_t=\mu_t+\gamma t+\varepsilon_t \tag{1}$$

where the drift term is written as:

$$\mu_t=c_0+\sum_{k=1}^{s}c_k\sin(2\pi kt/T)+\sum_{k=1}^{s}d_k\cos(2\pi kt/T),\quad s\leq T/2 \tag{2}$$

In the above formulation ε_t is a stationary disturbance term with variance σ_ε^2, s is the maximum number of frequencies, k is a particular frequency and T is the total number of observations. The drift term represents the Fourier approximation written as a deterministic function of sine and cosine terms. Note that by imposing $\alpha_k = \beta_k = 0$, one gets the constant mean or trend return specification. In contrast to other possible series expansions (e.g. Taylor series) the Fourier expansion has the advantage of acting as a global approximation (see Gallant, 1981). This property is obtained even if one specifies a small number of frequencies. In fact, Enders and Lee (2006) argue that a large value of s in a regression framework uses many of the degrees of freedom and leads to an over-fitting problem.

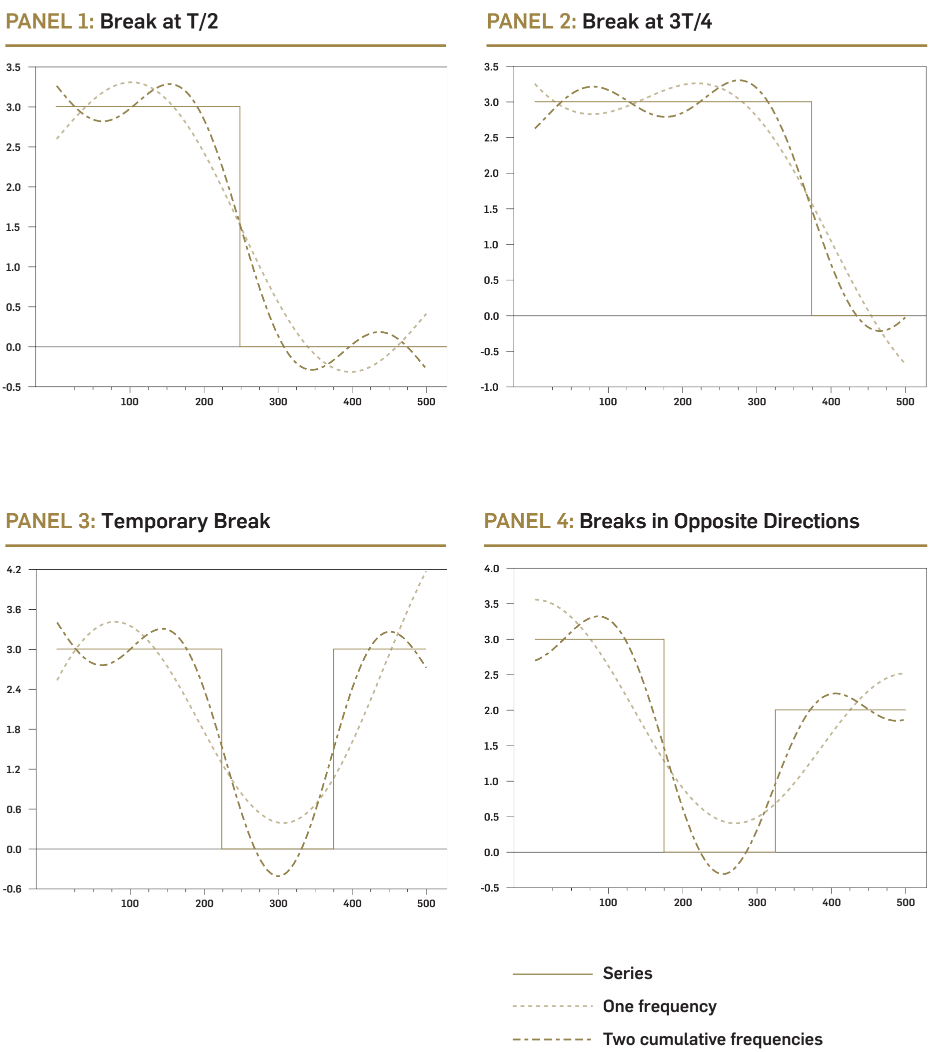
To illustrate the approximation properties of a Fourier series, the paper considers first a single frequency in the Data Generating Process (DGP):

$$\mu_t=c_0+c_k\sin(2\pi kt/T)+d_k\cos(2\pi kt/T) \tag{3}$$

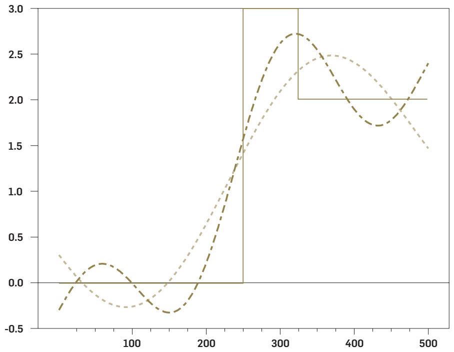
where k is the single frequency selected in the approximation, and c_k and d_k represent the magnitudes of the sinusoidal terms.

This study considers several possible patterns for the occurrence of a break. Thus, for T=500, the paper simulates one break, two breaks, and trend breaks both in the middle and towards the extremes. The paper illustrates the cases for temporary, permanent, and reinforcing breaks. We display the results below in Panels 1 through 9 (i.e., Figure 1). As in Enders and Lee (2006), Panels 1 and 2 illustrate approximations for breaks towards the end of a series. In Panel 3 the series has a temporary, though long-lasting break. Panels 4 and 5 display permanent breaks in opposite directions while in Panel 6 the breaks are in the same direction. Finally, Panels 7-9 depict breaks in the intercept and slope of a trending series. The paper estimates the coefficients of the sinusoidal terms by performing a simple regression of y_t on μ_t and a time trend.

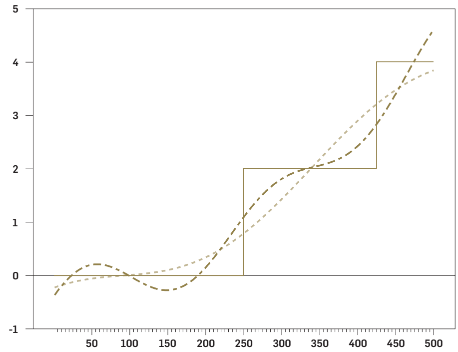
Figure 1. Approximation of Structural Breaks with Fourier Transforms



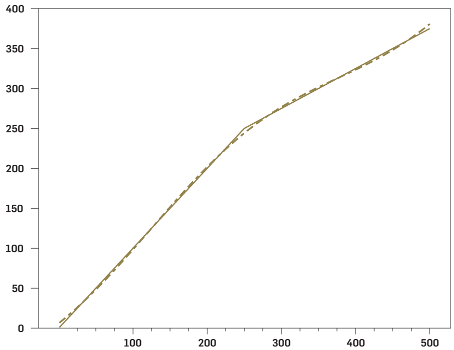
PANEL 5: Short Breaks in Opposite Directions



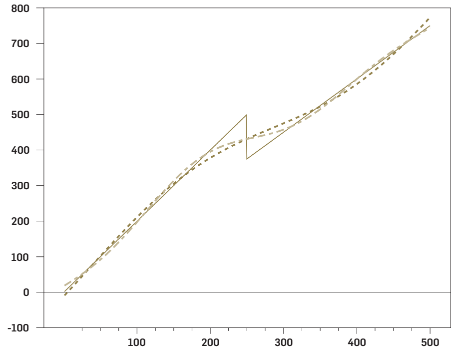
PANEL 6: Reinforcing Breaks



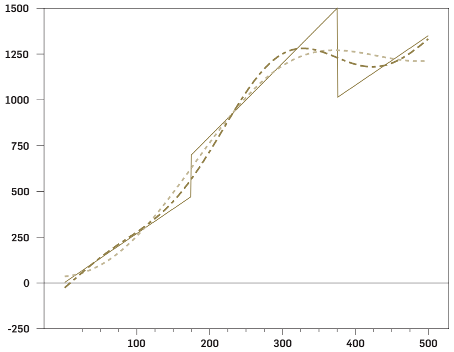
PANEL 7: Trend Breaks



PANEL 8: Change in Level Slope



PANEL 9: Temporary Change in Level Slope



Series
One frequency
Two cumulative frequencies

One can draw several conclusions based on the visual inspection of the graphs. First, a single frequency k=1 or two cumulative frequencies n=2, can approximate a large variety of breaks. Second, the Fourier transform approximates well even when the breaks are asymmetric (see Panels 1 and 2). Third, a Fourier series works best when the break is smooth over time which means it may not be suited for abrupt and sharp breaks of short duration (see Panel 5). An additional frequency of k=2 can improve the fit in this situation. Interested readers are referred to Enders and Lee (2006) and Becker et al. (2006) who have a longer discussion on the properties of the Fourier approximations. The next section introduces a new model to approximate long-run volatility.

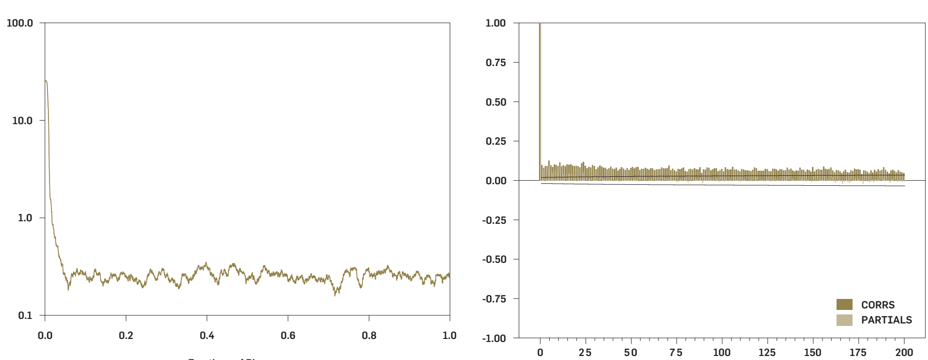
3. A New Model for Unconditional Volatility

As the introductory part suggested, the simple GARCH(1,1) may not be appropriate because it implies a long-run level of the volatility that is constant. However previous research regarding the presence of various shifts in stock returns suggests that structural changes in the second moment induce global nonstationarity. This invalidates the use of the simple GARCH(1,1). It is known that breaks shift the spectral density function towards frequency zero. This indicates that the frequencies to be used are towards the low end of the spectrum (see Enders and Lee, 2006). A simple visual inspection of the autocorrelation function and periodogram of absolute returns of S&P 500 confirms this fact:

Figure 2. S&P 500

Sample Spectrum of absolute returns of S&P 500

Sample ACF of returns of S&P 500



As you can note from the top graph, the most important frequencies that have an impact on the absolute returns are at the low end of the sample spectrum which is indicative of structural breaks. Both graphs confirm the presence of long memory in financial returns - slow decay with lags still significant at the 200th lag. These findings suggest the use of the following model whose aim is to capture various unknown shifts in long-run volatility. The paper denotes it the basic Fourier-GARCH:

$$r_t=\mu+v_t\sqrt{u_t h_t},\quad \text{where}\quad v_t|I_{t-1}\sim iid(0,1) \tag{4}$$

$$h_t=(1-\alpha-\beta)+\alpha\left(\frac{(r_t-\mu)^2}{u_{t-1}}\right)+\beta h_{t-1} \tag{5}$$

$$\mu_t=exp\left[a_0+\sum_{k=1}^{s}\left(a_k\sin\frac{2\pi kt}{T}+b_k\cos\frac{2\pi kt}{T}\right)\right],\quad s\leq T/2 \tag{6}$$

The model preserves the parsimony of the GARCH(1,1) model while it allows the unconditional expectation of the volatility to be a function of time and of cycles of different frequencies. A simple extension allows the unconditional mean to be a function of time as well - higher unconditional variance certainly requires higher unconditional mean. The time varying first moment is also approximated using a Fourier representation:

$$\mu_t=c_0+\sum_{k=1}^{s}\left(c_k\sin\frac{2\pi kt}{T}+d_k\cos\frac{2\pi kt}{T}\right) \tag{7}$$

Given its flexible setup, the Fourier-GARCH captures both short and long-run dynamics. Note that:

$$E(r_t-\mu)^2=E(v_t^2u_th_t)=u_tE(h_t)=u_t \tag{8}$$

The study uses an exponential representation of the Fourier transform to ensure its positivity. Goodness of fit measures like the BIC or AIC criterions are employed to choose the proper number of frequencies exogenously. They are computed as follows:

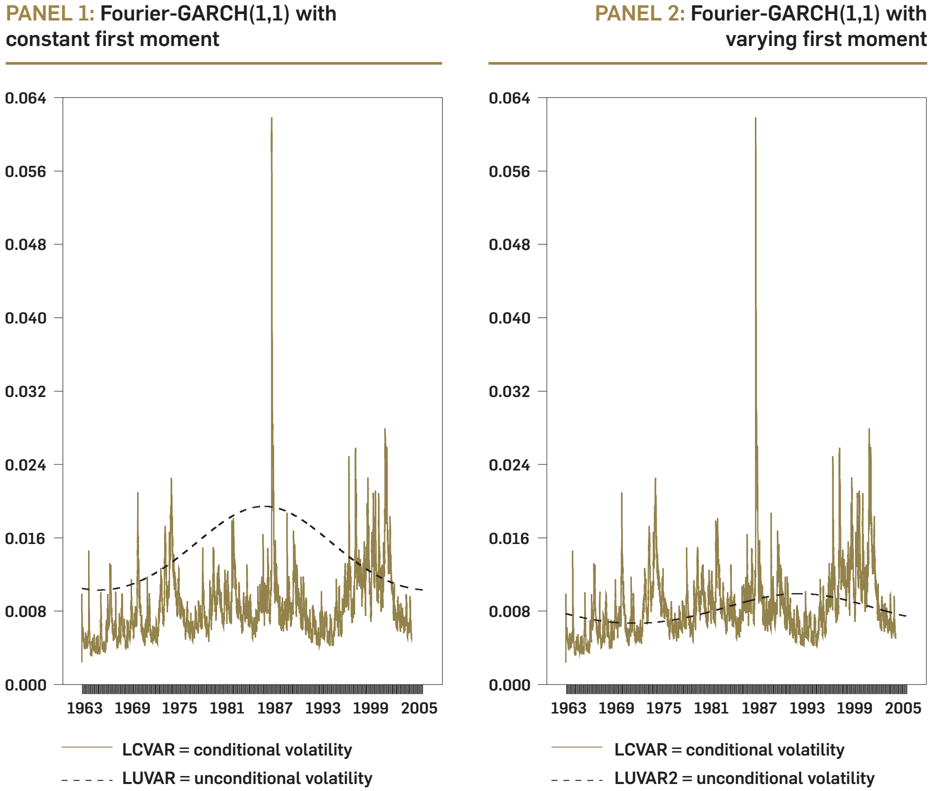
$$AIC=-lnL+2n,\quad L=-\sum_{t=1}^{T}\left[ln(h_tu_t)+\frac{(r-\mu)^2}{h_tu_t}\right] \tag{9}$$

$$BIC=-lnL+nln(T),\quad L=-\sum_{t=1}^{T}\left[ln(h_tu_t)+\frac{(r-\mu)^2}{h_tu_t}\right] \tag{10}$$

Here n denotes the number of parameters estimated by the model. The advantage of using the AIC and BIC criterions is that they include a penalty for the additional estimated parameters. Throughout the estimation the criterions employ only integer frequencies.

The advantage of using a time varying first moment for a sample of forty years of daily data of S&P 500 absolute returns is highlighted below:

Figure 3. Graphs of the Conditional and Unconditional Volatility of S&P 500



Note the better fit of the second model which augments the basic Fourier-GARCH representation with a time varying intercept as in equation (7). However, given the presumption that a higher long run volatility requires a higher long-run return, the paper proposes the Fourier-M model that includes the unconditional time-varying volatility in the equation for the mean:

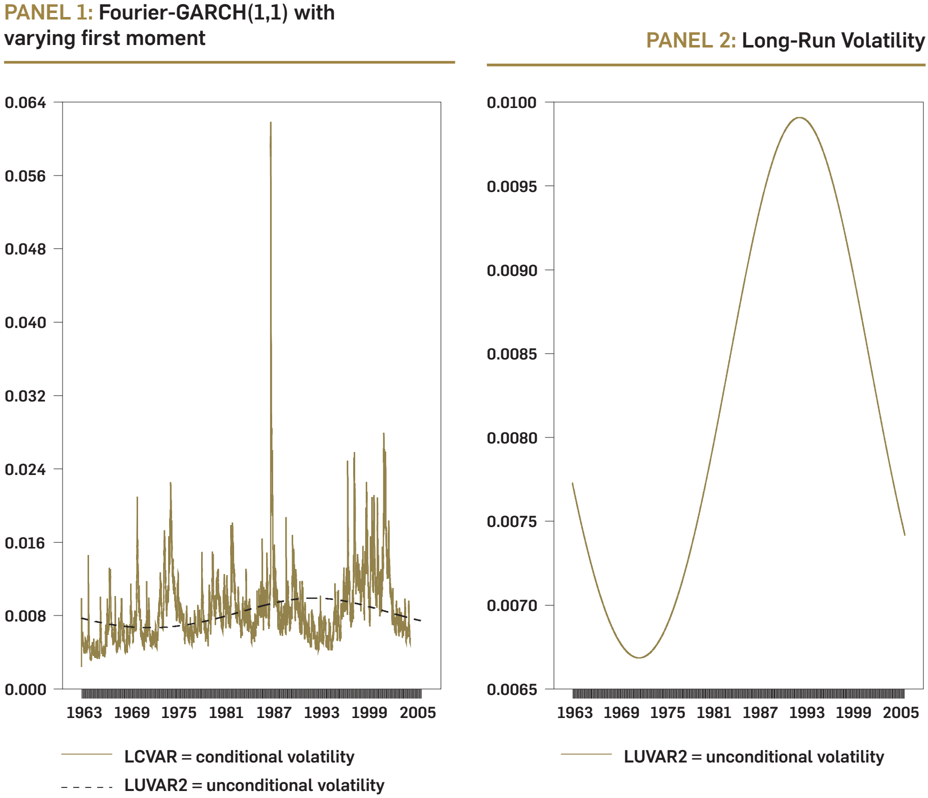
$$r_t = \gamma u_t + v_t \sqrt{u_t h_t}, \quad \text{where} \quad v_t | I_{t-1} \sim iid(0,1) \tag{11}$$

In this way, both the first and the second moment change over time while the underlying model ensures a parsimonious representation.

One way to assess the persistence or long memory in stock returns is to compute the sum of the slope coefficients in conditional volatility. If the sum is close to one, then conditional volatility is said to be almost integrated and it displays very slow time decay. However the support for long-memory is weakened if one finds that a changing first and/or second moment is responsible for the persistence effect. If the sum of the coefficients is significantly less than one after one accounts for shifts in the unconditional mean or volatility, then one can conclude that the volatility process is stationary but suffers from structural shifts (see Perron and Qu, 2007).

A sample of daily returns on S&P 500 from 1963:01:02 to 2005:2:30 illustrates this discussion. The best representation is the one that specifies a single frequency both for the mean and for the unconditional volatility:

Figure 4. **Graphs of the Conditional and Unconditional Volatility of S&P 500**



Note the slow and gradual increase of long run volatility from the 1960's until the 1980's. Also note that the estimated long run volatility of the 1990's is lower than the one for previous decades, which is consistent with market facts.

4. Model Validation and Persistence Effects

The paper uses several representative stocks of S&P 500 to assess the long memory effect of stock returns using the new models. The first 12 stocks of the index are selected according to their market percentage participation as of March 2005. Table 1 shows their ticker, sector classification and percent of total assets.

Table 1. **Market Capitalization of 13 Companies on S&P 500 as of 2/28/2006**

Ticker	Issue Name	Sector	% of Total Assets
XOM	Exxon Mobil Corp.	Energy	3.19
GE	General Electric Co.	Industrials	3
MSFT	Microsoft Corp.	Industrials	2.12
C	Citigroup Inc.	Financials	2.03
PG	Procter & Gamble	Consumer Staples	1.73
PFE	Pfizer Inc.	Health Care	1.67
AIG	American Intl. Group Inc.	Financials	1.49
JNJ	Johnson & Johnson	Health Care	1.48
MO	Altria Group Inc.	Consumer Staples	1.29
CVX	Chevron Corp. New	Energy	1.09
IBM	International Business Mach.	Information technology	1.09
INTC	Intel Corp.	Information technology	1.07

The data has been obtained from the Center of Research in Security Prices made available through the WRDS database. The longest sample period available is 1926:01:02 - 2005:12:30 and corresponds to Exxon, IBM, Chevron, Phillip-Morris and General Electric. Other stock returns have shorter sample periods (i.e. Procter & Gamble from 1929:01:02 onwards, Pfizer and Johnson & Johnson start in 1944; Intel from 1972, while the rest start in 1986). For each stock return, the study chooses exogenously an integer or cumulative frequencies according to the AIC and BIC criterions. According to Enders and Lee (2006), a frequency greater than 5 uses many of the degrees of freedom and leads to an over-fitting problem.

Table 2 displays the results from applying the AIC and BIC criterions to identify the best in sample fitting model. The above mentioned criterions indicate that in most cases the best representation is the basic Fourier-GARCH(1,1) model. The coefficients of the sine and cosine terms with up to 5 frequencies are significant at the 5% level both for the basic and for the extended models. However, given that in the model for the mean each additional frequency requires the estimation of two more coefficients,

Table 2. **AIC, BIC, and the Log-Likelihood**

A

	AIG			Chevron			Citigroup			Exxon			General Electric		
Frequencies	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)
1	0.989	40.287	11.011	0.044	47.720	11.956	1.609	40.512	10.391	0.003	47.679	11.997	0.048	47.724	11.952
2	4.987	61.217	11.013	4.035	67.603	11.965	5.634	57.505	10.386	4.005	67.574	11.995	4.076	67.643	11.924
3	8.987	79.275	11.013	8-034	87.495	11.965	9.617	74.456	10.383	8.005	87.465	11.995	8.046	87.506	11.954
4	12.987	97.332	11.013	12.035	107.387	11.965	13.614	91.420	10.387	12.005	107.357	11.995	12.047	107.398	11.953
5	16.987	115.390	11.013	16.035	127.278	11.965	17.615	108.389	10.385	16.005	127.249	11.995	16.046	127.290	11.954
1 (mean shifts)	5.132	61.363	10.868	4.645	68.213	11.354	5.936	57.807	10.064	5.488	69.056	10.512	4.375	67.943	11.625
1 (Fourier-M)	1.002	43.174	10.985	0.039	47.714	11.961	4.757	43.660	7.243	0.008	47.683	11.992	0.074	47.750	11.926

B

	IBM			Intel			Johnson & Johnson			Microsoft			Pfizer		
Frequencies	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)	AIC	BIC	(l)
1	0.028	47.704	11.972	1.142	48.818	10.858	0.334	46.290	11.666	1.532	40.631	10.468	0.361	46.396	11.639
2	4.026	65.594	11.974	5.153	61.383	10.847	4.334	65.609	11.666	5.629	57.762	10.371	4.379	65.759	11.621
3	8.027	87.487	11.973	9.153	79.441	10.847	8.334	84.927	11.666	9.615	74.558	10.385	8.362	86.087	11.638
4	12.026	107.378	11.974	13.152	97.498	10.848	12.334	104.246	11.666	13.507	91.706	10.493	12.361	104.431	11.639
5	16.026	127.270	11.974	17.150	115.554	10.850	16.333	123.565	11.666	17.539	108.771	10.461	16.360	123.775	11.639
1 (mean shifts)	4.930	68.498	11.070	5.366	61.597	10.634	5.257	66.532	10.743	5.696	57.829	10.304	6.810	68.190	9.190
1 (Fourier-M)	0.029	47.705	11.971	1.162	43.335	10.838	0.335	46.291	11.665	1.494	40.593	10.506	0.364	46.399	11.635

C

	Phillip-Morris			Procter & Gamble		
Frequencies	AIC	BIC	(l)	AIC	BIC	(l)
1	0.074	47.750	11.926	0.043	47.718	11.957
2	4.077	67.645	11.923	4.041	67.240	11.959
3	8.076	87.537	11.924	8.041	87.039	11.959
4	12.077	107.428	11.923	12.041	106.839	11.959
5	16.077	127.320	11.924	16.041	126.639	11.959
1 (mean shifts)	4.611	68.179	11.389	4.962	68.161	11.038
1 (Fourier-M)	0.079	47.755	10.921	0.047	47.446	11.953

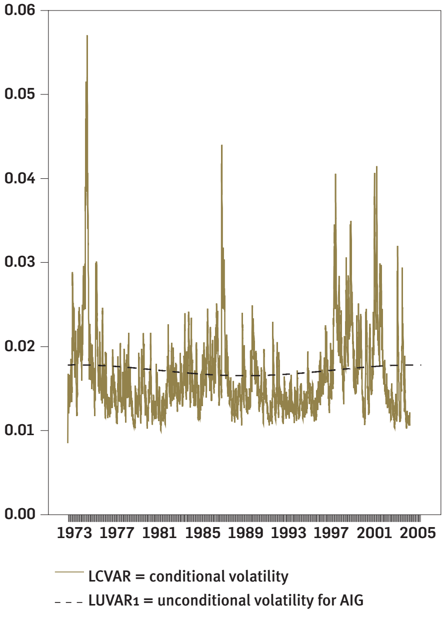
the additional penalty increases the values of the AIC and BIC criterions relative to the ones for the basic model. This is not surprising given that the BIC criterion favors more parsimonious representations. Several exceptions to the finding above are noteworthy. In the case of Microsoft for instance, both criterions select the Fourier-M model to be the optimal representation. Also, the Fourier-M model gives the best fit for Chevron as well. Note that the basic Fourier-GARCH(1,1) and the Fourier-M models have very close values for the BIC and SBC criterions. This is true because they estimate the same number of parameters (i.e. six coefficients). In rest, the increased penalty due to the additional coefficients that are estimated in the models with two or more cumulative frequencies is greater than the better fit that is obtained. Therefore, the single frequency representation fits the data best for all models. Figures 5 through 7 show several graphs of the conditional and long-run volatilities obtained using both a constant and a time varying first moment. Note that for all series the long run volatility changes smoothly over time.

Figures 5, 6 and 7.

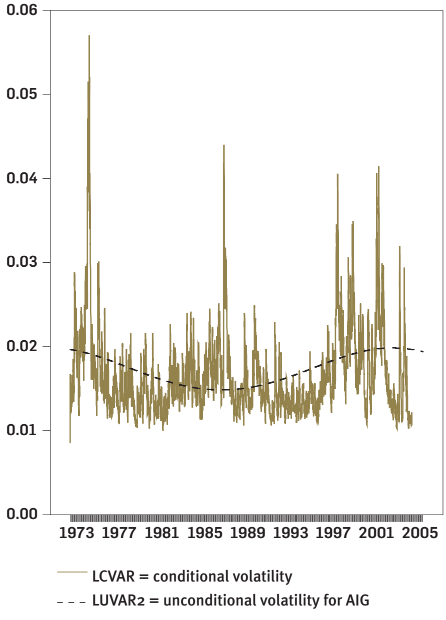
Conditional and Unconditional Volatility from Fourier-GARCH(1,1)

5(a) AIG:

PANEL 1: Fourier-GARCH(1,1) with constant first moment

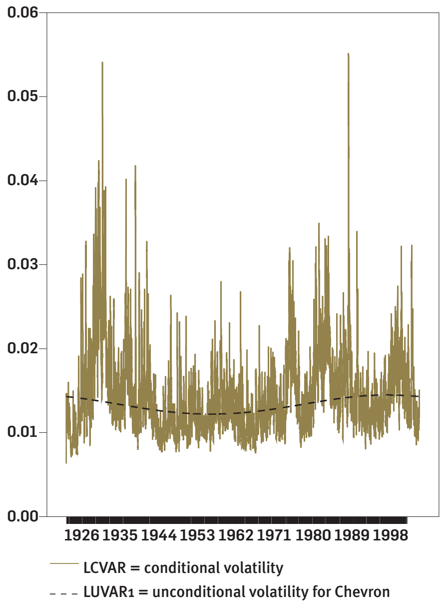


PANEL 2: Fourier-GARCH(1,1) with varying first moment

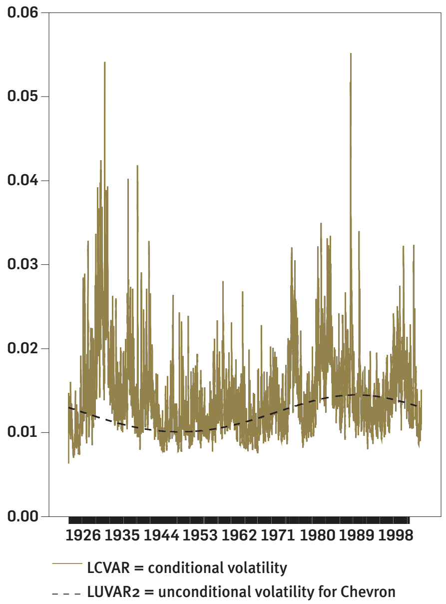


5(b) Chevron:

PANEL 1: Fourier-GARCH(1,1) with constant first moment

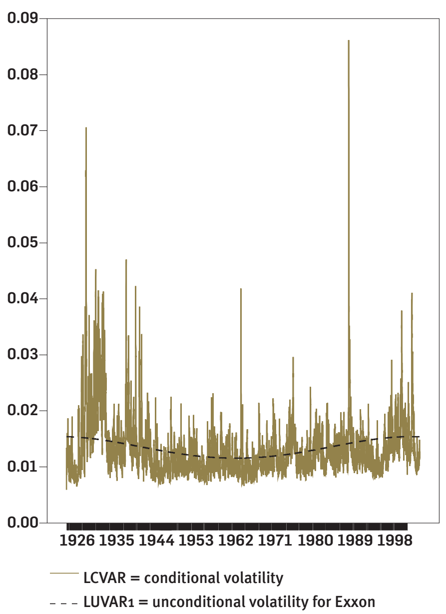


PANEL 2: Fourier-GARCH(1,1) with varying first moment

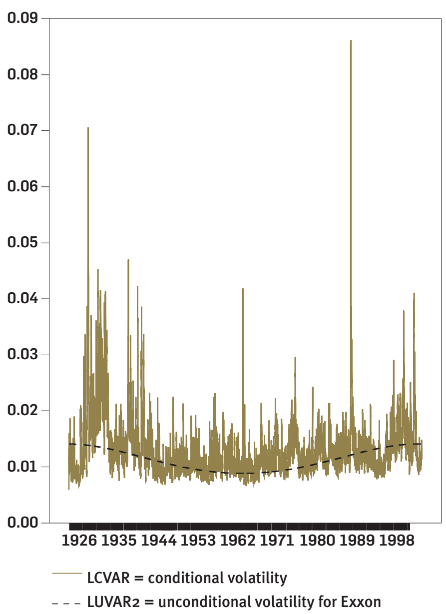


6(a) Exxon:

PANEL 1: Fourier-GARCH(1,1) with constant first moment

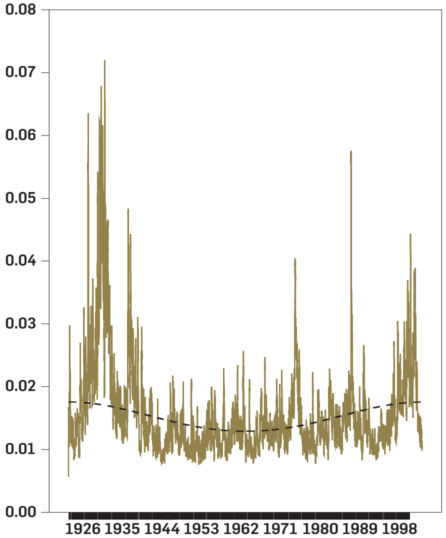


PANEL 2: Fourier-GARCH(1,1) with varying first moment



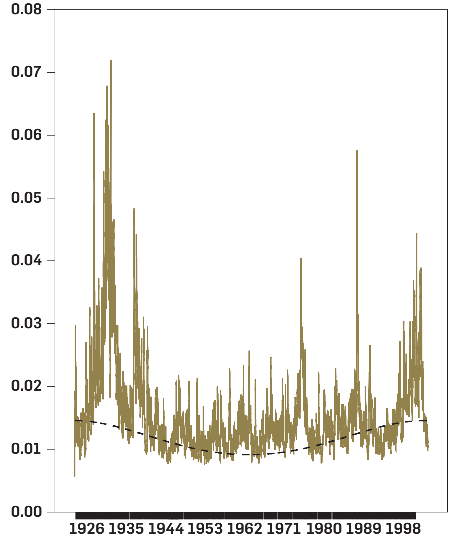
6(b) GE:

PANEL 1: Fourier-GARCH(1,1) with constant first moment



LCVAR = conditional volatility
- - - LUVAR1 = unconditional volatility for GE

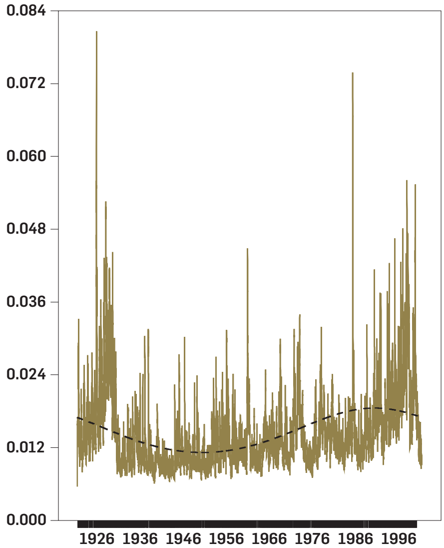
PANEL 2: Fourier-GARCH(1,1) with varying first moment



LCVAR = conditional volatility
- - - LUVAR2 = unconditional volatility for GE

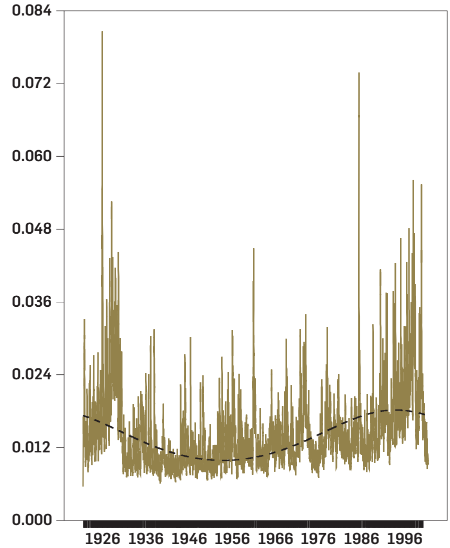
7(a) IBM:

PANEL 1: Fourier-GARCH(1,1) with constant first moment



LCVAR = conditional volatility
- - - LUVAR1 = unconditional volatility for IBM

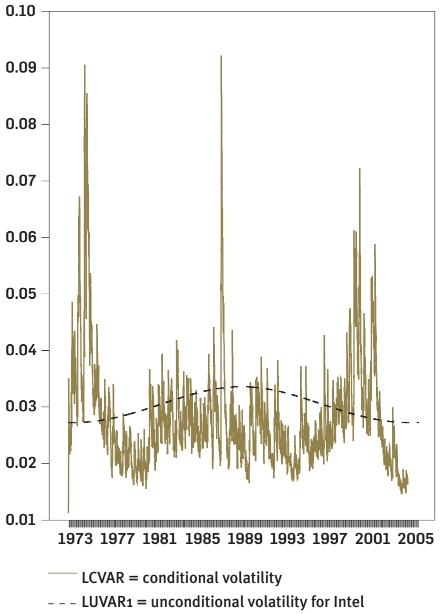
PANEL 2: Fourier-GARCH(1,1) with varying first moment



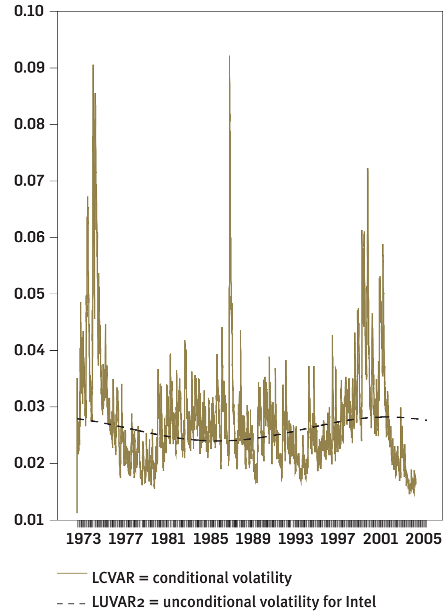
LCVAR = conditional volatility
- - - LUVAR2 = unconditional volatility for IBM

7(b) Intel:

PANEL 1: **Fourier-GARCH(1,1) with constant first moment**



PANEL 2: **Fourier-GARCH(1,1) with varying first moment**



Next the paper investigates whether the selected returns display the long memory property that is usually observed in financial data. To this end, the study estimates four competing models:

- the common GARCH(1,1) developed by Bollerslev (1987) denoted M_0 ;
- the basic Fourier-GARCH(1,1) with constant first moment, denoted M_1 ;
- the Fourier-GARCH(1,1) with a time varying first moment, denoted M_2 ;
- the Fourier-M(1,1) with long-run volatility in the mean, denoted M_3 .

Table 3 shows the results. Clearly, model M_2 provides the best reduction of the persistence effect for most series. For 10 of the 12 stock returns considered, the long memory effect is dramatically reduced in many instances by half or even more (i.e. GE, Pfizer, IBM, Phillips-Morris, Chevron, Intel, Procter & Gamble, Exxon, Johnson & Johnson, and Citigroup).

Table 3. **Persistence of financial volatility**

	M_0: GARCH(1,1)	M_1: Fourier-GARCH(1,1) with constant mean	M_2: Fourier-GARCH(1,1) with time-varying mean	M_3: Fourier-GARCH(1,1)
Companies	$\alpha+\beta$	$\alpha+\beta$	$\alpha+\beta$	$\alpha+\beta$
AIG	0.98024	0.98034	0.97753	0.96798
Chevron	0.98704	0.98373	0.75108	0.96577
Citigroup	1.00104	0.98388	0.57667	0.99360
Exxon	0.98333	0.95727	0.54307	0.95894
General Electric	0.99256	0.99013	0.80713	0.99261
IBM	0.99180	0.96291	0.51090	0.95930
Intel	0.99185	0.99206	0.64818	0.98175
Johnson&Johnson	0.95222	0.88250	0.01595	0.90133
Microsoft	0.06820	0.10247	0.22565	0.09550
Pfizer	0.97707	0.90421	0.40066	0.85240
Phillip-Morris	0.99877	0.99251	0.75108	0.98887
Procter&Gamble	0.99595	0.96786	0.29293	0.96698

Note that the basic representation (i.e. the M_1 model above) has only little impact on overall persistence in the short-run volatility. In most cases, its persistence is only slightly lower than the one of the GARCH(1,1) representation.

This is surprising given that this model gives the best fit according the AIC and BIC criterions in 10 out of the 12 stocks considered. Note that model M_3 clearly outperforms model M_1 in terms of reduced long memory effect as well. The main conclusion is that allowing only for the second moment to vary over time is not enough to account for the strong persistence effect observed in financial returns. However, in contrast to the basic model, a time-varying first moment in the equation for the mean reduces significantly the persistence in short run volatility.

5. Conclusion

The paper proposes a new model to estimate the short and long run dynamics in financial data that takes into account the possibility of a time varying first and second moment. The Flexible Fourier transform of Gallant (1981) approximates the unknown date and shape of any structural break in the first and second moment as smooth processes. The study shows that Fourier series are able to approximate a wide variety of breaks of an unknown form. The basic Fourier-GARCH representation modifies the popular GARCH(1,1) to include a time varying unconditional variance. The paper proposes two extensions to the basic model. The first extension specifies a time varying first moment, while the second extension includes the long-run volatility in the equation for the mean. The results suggest that persistence still remains significant in the short run volatility for the basic model. However, the so called long memory effect disappears if one includes a time varying first moment in the model for the mean. This suggests that conditional volatility persistence is an artifact of the misspecification of the model for the mean.

References

- Becker, R., Enders, W. and Hurn, S. (2001). Modelling structural change in money demand using a Fourier-series approximation, Research Paper Series 67, Quantitative Finance Research Centre, University of Technology, Sydney.
- Becker, R., Enders, W. and Lee, J. (2006). A stationarity test in the presence of an unknwon number of smooth breaks, *Journal of Time Series Analysis,* **27**, pp. 381–409.
- Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return, *The Review of Economics and Statistics,* **69**, pp. 542–47.
- Diebold, F. X. and Inoue, A. (2001). Long memory and regime switching, *Journal of Econometrics,* **105**, pp. 131–159.
- Enders, W. and Lee, J. (2006). Testing for a unit root with a nonlinear Fourier function. Mimeo, University of Alabama.
- Engle, R. and Bollerslev, T. (1986). Modeling the persistence of conditional variances, *Econometric Reviews*, **5**, pp. 1–50.
- Engle, R. F. and Rangel, J. G. (2008). The spline-garch model for low-frequency volatility and its global macroeconomic causes, *Review of Financial Studies*, **21**, pp. 1187–1222.
- Gallant, R. (1981). On the basis in flexible functional form and an essentially unbiased form: the flexible fourier form, *Journal of Econometrics*, **15**, pp. 211–353.
- Granger, C. W. J. (1980). Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics*, **14**, pp. 227–238.
- Granger, C. W. J. and Joyeux, R. (1980). An introduction to long memory time series and fractional differencing, *Journal of Time Series Analysis*, **1**, pp. 15–30.
- Granger, C. W. J. and Joyeux, R. (1986). Modeling the persistence of the conditional variances: a comment, *Econometric Reviews*, **5**, pp. 51–56.

- Kapetianos, G., Shin, Y. and Snell, A. (2003). Testing for a unit root in the nonlinear STAR framework, *Journal of Econometrics*, **112**, pp. 359–379.
- Lamoureux, C. G. and Lastrapes, W. D. (1990). Persistence in variance, structural change, and the GARCH model, *Journal of Business & Economic Statistics*, **8**(2), pp. 225–235
- Leybourne, S., Newbold, P. and Vougas, D. (1998). Unit roots and smooth transitions, *Journal of Time Series Analysis*, **19**, pp. 83–97.
- Maddala, G. and Kim, I.-M. (1998). *Unit roots, cointegration and structural change*, Cambridge University Press, Cambridge (UK).
- Mikosch, T. and Starica, C. (2004). Non-stationarities in financial time series, the long range dependence and the IGARCH effects, Econometrics 0412005, EconWPA.
- Perron, P. and Qu, Z. (2007). An analytical evaluation of the log-periodogram estimate in the presence of level shifts. Available at http://people.bu.edu/perron/.
- Starica, C. and Granger, C. (2005). Nonstationarities in stock returns, *The Review of Economics and Statistics*, **87**, pp. 503–522.