

Firms' Accruals and Tobin's q

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Abstract

According to the neoclassical theory of investment, if firms' accruals are a form of short-term investment they should be greatly influenced by the shadow price of capital, namely Tobin's q . In the presence of financial market imperfections, cash-flows should also impact accruals since they proxy for liquidity constraints. In this paper, we test a new version of the cash-flows augmented accrual model featuring a proxy for Tobin's q , and compare it to the most common models found in the literature. To deal with the measurement errors often encountered in accounting data and Tobin's q empirical proxies, we rely on a modified version of the Hausman artificial regression, and find that all the key parameters of the accrual models are indeed systematically biased with measurement errors. More importantly, our findings largely qualify the accrual investment perspective, as both cash-flows and Tobin's q are found strongly significant regressors of firms' accruals. Interestingly, we find that the Tobin's q augmented model is able to isolate discretionary accruals, and to deliver residuals quite close to zero on average.

Keywords:

Discretionary accruals, Investment, Tobin's q , Cash-flows, Measurement errors, Instrumental variable estimators.

JEL classification:

M41, C12, D92.

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Los ajustes por devengo de las empresas y la q de Tobin

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Resumen

De acuerdo con la teoría neoclásica de la inversión, si los ajustes por devengo de las empresas constituyen una forma de inversión a corto plazo, deberían estar fuertemente influenciados por el precio sombra del capital, denominado q de Tobin. En presencia de imperfecciones de mercado, los flujos de caja también deberían influenciar los ajustes por devengo ya que son una variable proxy de las restricciones de liquidez. En este artículo se contrasta una nueva versión del modelo de ajustes por devengo aumentado con flujos de caja con una variable proxy para la q de Tobin, el cual se compara con los modelos que habitualmente se pueden encontrar en la literatura. Para tratar la cuestión de los errores de medida que a menudo se encuentran cuando se trabaja con datos contables y variables proxy empíricas de la q de Tobin, nos apoyamos en una versión modificada de la regresión artificial de Hausman, encontrando que todos los parámetros clave de los modelos de ajuste por devengo están sistemáticamente sesgados debido a los errores de medida. Más importante aún es el hecho de que nuestros resultados sostienen en gran medida la perspectiva de los intereses devengados, ya que tanto los flujos de caja como la q de Tobin son regresores altamente significativos de los ajustes por devengo de las empresas. También resulta interesante el hecho de que el modelo aumentado de la q de Tobin es capaz de aislar los ajustes por devengo discrecionales, y que los residuos de la ecuación se adaptan muy bien a la predicción de los rendimientos de las acciones.

Palabras clave:

Ajustes por devengo discrecionales, inversión, q de Tobin, flujos de caja, errores de medida, estimadores de variables instrumentales.

■ 1. Introduction

A fundamental drawback of cash-flows is that they present timing and matching problems that cause them to be a very noisy measure of firm performance. To mitigate these issues, it is common to rely on accounting accruals to intertemporally smooth earnings. Accruals are then used to separate the timing of cash flows from their accounting recognition. Based on the cash-flows statement, total accruals, the sum of discretionary and non-discretionary accruals, are defined as the difference between firm earnings and cash-flows (Jones, 1991; Bartov *et al.*, 2001), and, as such, are related to firm performance. For this reason, accrual models are widely used by financial analysts to assess the level of *discretionary* accruals, a significant predictor of stocks returns¹ (Sloan, 1996; Dechow and Dichev, 2002; Fama and French, 2007; Hirshleifer *et al.*, 2009). Actually, one of the main reasons why accrual models have generated such attention in the finance literature is precisely the fact that the accrual equations residuals carry valuable information about stock returns. This relates to the famous accrual anomaly (Sloan, 1996; Dechow and Dichev, 2002), which, contrary to the asset growth and profitability anomalies, seems to be very robust (Fama and French, 2007). To better isolate the nonlinear relationship between accruals and firm performance, researchers often include proxies for economic values and investment potential such as property plant and equipment (*PPE*) and sales. However, the literature is almost mute about the investment perspective of accruals, even though accruals measure investment in working capital. According to this perspective, the standard specification of the accrual models misses some important aspects of accruals, namely the fact that accruals constitute a form of short-term investment, at least in terms of working capital. In particular, the accrual anomaly might actually relate to the investment information embedded in accruals. For example, Wu *et al.* (2007, 2010) show that the negative relationship between accruals and the discount rate helps explain the accruals anomaly.

In this paper, our motivation is to build an accrual model consistent with the neoclassical theory of investment to study the positive relationship between accruals and investment variables. In the literature, the standard models used to estimate non-discretionary accruals generally rely on OLS estimation (e.g., Jones, 1991; Bartov *et al.*, 2001; Xie, 2001, Kothari *et al.*, 2005; Wu *et al.*, 2007, 2010 among many others). It is also common to use models taking into account various aspects of simultaneity biases and address the problem of measurement errors associated with accruals (e.g. Kang and Sivaramakrishnan, 1995; Hansen, 1999; Young, 1999; Hribar and Collins, 2002; Zhang, 2007; Ibrahim, 2009). To control for firm performance, many studies include a variable such as *ROA*, since accruals and performance are positively related (e.g., Kothari *et al.* 2005). By contrast, in this paper, we focus on two commonly used accrual models, the

¹ For an extensive survey on the relationship between accounting information and capital markets efficiency see Kothari (2001).

Jones (1991) model, our benchmark, and the cash-flows augmented Jones model (Dechow, 1994; Dechow and Dichev, 2002; Hirshleifer *et al.*, 2004; Zhang, 2007; Hirshleifer *et al.*, 2009) to compare them with a new type of model where firms' accruals are explicitly specified as short-term investment. According to the neoclassical theory, Tobin's q (the shadow price of capital) is a key explanatory variable of investment (e.g. Abel and Blanchard, 1983; Fazzari *et al.*, 1988; Blanchard and Fischer, 1989; Gilchrist and Himmelberg, 1995; or Brown and Petersen, 2009 and Brown *et al.*, 2009). In this sense, if accruals are indeed a form of short-term investment, they should be influenced by Tobin's q . Relatedly, our approach aims at rigorously justifying the introduction of cash-flows in the accrual models, investment theory predicting that cash-flows proxy for firms' liquidity constraints. The main contribution of this paper is thus to propose a model where accruals are a function of investment variables.

Relatedly, the inclusion of Tobin's q adds to the endogeneity problem often encountered in accrual models, since this variable is notoriously plagued with measurement errors, a well documented fact in the investment literature (e.g., Hayashi, 1982; Erickson and Withed, 2000, 2002, among many others). Furthermore, when examining non-discretionary and discretionary accruals – the latter being the error term in the accrual models – we have to bear in mind the fact that this error term is the portion of accruals managed by firms, so that it may sometimes be influenced by various earnings management practices (Dechow *et al.*, 1995; Burghstahler and Dichev, 1997; DeFond and Park, 1997; DeGeorge, Patel and Zeckhauser, 1999; Peasnell *et al.*, 2000; Xie, 2001; Leuz, Nanda and Wysocky, 2003; Marquardt and Wiedman, 2004; Hirshleifer *et al.*, 2004; Roychowdhury, 2006). This may result in a statistical anomaly worth detecting, as discretionary accruals are often used to forecast market returns. The accrual models sometimes account for heteroskedasticity with a form of weighted least-squares, but the measurement errors inherent to accounting data are usually assumed to be systematically biased in the same direction, even though they can cause a serious bias in the estimation if the orthogonality between the explanatory variables and the equation innovation is not satisfied. Rigorously correcting for measurement errors is thus imperative, and it is much desirable to resort to a robust estimation method in order to compute the accruals with the greatest possible accuracy (Ibrahim, 2009). To handle this task, we introduce new instruments based on a weighted optimal matrix of the higher moments of the explanatory variables, and apply these optimal instruments to an Hausman artificial regression (our Haus-C method).

Our results suggest that measurement errors have indeed a great influence on the parameters estimation of the basic Jones model. More precisely, our estimation of the accrual models confirms that important measurements errors contaminate the accounting measures of changes in sales and fixed assets, the two main explanatory

variables of the Jones model. More importantly, our main finding suggests that, Tobin's q , which has already a very significant positive impact on non-discretionary accruals when using the standard OLS method, displays an increased explanatory power when applying our Haus-C procedure. The Haus-C procedure delivers a coefficient of the error adjustment regressor comparable, in level, to the Tobin's q coefficient itself. We can interpret this result as new evidence that firms' expectations are partly incorporated in future cash-flows, a point often mentioned in the empirical literature on investment. Relatedly, when we introduce Tobin's q in the accruals equation, the cash-flow variable has a smaller influence on short-term investment. When measurement errors are properly accounted for, the role of Tobin's q is reinforced, while cash-flows seem to play a minor role, although non-trivial. Overall, the evidence we gather tends to support the empirical literature on firm investment, and in particular the idea that market imperfections also impend the Modigliani and Miller theorem to hold for short-term investment. In the context of accruals, these imperfections are likely associated with the liquidity constraints in earnings management and the preference for directly self-financing accruals with cash-flows before resorting to external finance.

This paper is organized as follows. In section 2 we present the theoretical underpinning of our approach, based on the neoclassical theory of investment, and describe the three accrual models we analyze, along with some considerations regarding measurement errors. In section 3 we detail the empirical results, and in section 4 we compare the residuals of our accrual equations to assess the performance of the Tobin's q augmented model. Section 5 concludes.

■ 2. The Model

With a balance sheet approach, total accruals are defined as:

$$TA \equiv (\Delta CA - \Delta CASH) - (\Delta CL - \Delta STD - \Delta TP) - DEP \quad (1)$$

where ΔCA stands for change in current assets; $\Delta CASH$, change in cash or cash equivalents; ΔCL , change in current liabilities; ΔSTD , change in debt included in current liabilities; ΔTP , change in income taxes payable; and DEP represents the depreciation and amortization expenses. Note however that the negative depreciation term in the traditional accruals equation tends to strongly influence the model's fit, as the depreciation to assets ratio is five times higher than the accounts receivable and accounts payable, on average (Barth, Cram and Nelson, 2001). One way to deal with this issue is to look at short-term accruals and omit the long-run component, i.e., the depreciation (Teoh *et al.*, 1998a, b). Since this study focuses on the short-term investment dimension

of firms' accruals (the working capital component of accruals), we thus consider an alternative construct, CA which eliminates total depreciation from Equation (1).

2.1. Theoretical underpinning

To cast accruals in terms of short-term investment we resort to the neoclassical theory of investment, and in particular to the q theory of investment (Abel and Blanchard, 1983; Blanchard and Fischer, 1989; Adda and Cooper, 2003) which predicts that only one variable impacts investment, I , namely the shadow price of capital, Tobin's q , a variable incorporating all the relevant information related to the investment decision. In theory, investment is expressed as:

$$\frac{I_{it}}{k_{it}} = \varphi(q_{it}) \quad (2)$$

Where the i subscript refers to an individual firm or plant, and the t subscript represents time, whereas k_{it} is the firm's capital stock. This equation is the result of an intertemporal optimization program based on the maximization of the utility of a representative consumer subject to a resource constraint (Abel and Blanchard, 1983). In Equation (2), q accounts for both the gross return and the cost of capital, and is simply defined as the market value of capital over its replacement cost (Tobin, 1969). More precisely, in a general equilibrium setting *à la* Ramsey, this variable is the shadow price of capital, which is equal to the present discounted value of capital future marginal products. At the margin, the central planner who computes the optimization program equates the value of an additional unit of capital with its marginal cost, which increases with the rate of investment. The main implication of the theory is then: $\varphi'(q_{it}) > 0$.

To test it, Equation (2) is usually linearized as follows:

$$\frac{I_{it}}{A_{it}} = a_0 + a_1 q_{it} \quad (3)$$

the coefficient a_1 being related to the agent discount rate and to the adjustment costs of capital. Generally, researchers scale investment by assets instead of capital, which in our case is also more in line with the accruals accounting framework. Equation (3) is derived under the assumption of perfect markets. However, if markets are imperfect, financial imperfections and the liquidity constraints they entail may impact investment. In the context of market imperfections and credit market frictions, it is thus common to augment Equation (3) with a vector \mathbf{X}_t of control variables²:

$$\frac{I_{it}}{A_{it}} = a_0 + a_1 q_{it} + \mathbf{X}_t \theta \quad (4)$$

² As explained in the following section, nominal variables in the \mathbf{X} vector are also scaled by assets.

In addition to Tobin's q , in many studies, measures of cash flows are found significant factors influencing investment (e.g. Gilchrist and Himmelberg, 1995; Calmès, 2004, Brown *et al.*, 2009; Brown and Petersen, 2009). If market imperfections are at work, the relationship between investment and cash-flows ought to be positive since external finance is more costly than internal finance. Measures of profits may be also added to the control variables set to account for liquidity constraints. This generalization is still much debated in the literature because it is obviously at odds with the Modigliani and Miller (1958) theorems on the independence between the firm's structure of capital and its value, a result based on perfect financial markets. In this paper, we introduce a new specification of firms' accruals based on this theoretical underpinning, directly derived from Equation (4), as discussed below.

2.2. The empirical framework

2.2.1. The Jones Model of Accruals

The Jones (1991) model, our primary benchmark (model 1), is the most popular accrual model in the accounting literature. It does not rely on any economic or financial theory, but is rather an empirical model which relates the components of accruals to their statistical determinants as follows:

$$\frac{TA_{it}}{A_{i,t-1}} = \alpha_s \left(\frac{1}{A_{i,t-1}} \right) + \beta_s \left(\frac{PPE_{it}}{A_{i,t-1}} \right) + \delta_s \left(\frac{\Delta REV_{it}}{A_{i,t-1}} \right) + \varepsilon_{it} = \frac{T\hat{A}_{it}}{A_{i,t-1}} + \varepsilon_{it} \quad (5)$$

where TA is total accruals; A , total assets; PPE , gross property plant and equipment at the end of year t ; and ΔREV represents revenues in year t less revenues in year $(t-1)$. As usually done in the literature, we scale all variables by $A_{i,t-1}$ to account for the heteroskedasticity which might be present in ε_{it} . This precaution also helps control for size effects. Equation (5) may be decomposed in two parts, the non-discretionary accruals component and the discretionary accruals one. The fitted value of the equation, $\frac{T\hat{A}_{it}}{A_{i,t-1}}$, represents the non-discretionary accruals, while the innovation, ε_{it} , is the discretionary part of accruals. Two control variables of the benchmark model relate, respectively, to the two main components of accruals, working capital and depreciation. The first control variable, ΔREV_{it} , often replaced by the change in sales in the literature (e.g., Cormier *et al.*, 2000), is associated with working capital, while the second control variable, PPE_{it} , is linked to depreciation. Usually, the ΔREV_{it} coefficient is found positively related to total accruals. Indeed, an increase in ΔREV_{it} should lead to an increase in working capital since accounts receivable are generally more sensitive to changes in sales than accounts payable³. Furthermore, the coefficient of PPE_{it} should be negative as PPE_{it} determines the depreciation expenses, a negative component of accruals. Note however that there is potentially an endogeneity issue

³ In some cases, the sign of this coefficient may be negative. For more details, see McNichols and Wilson (1988).

here because the control variable PPE_{it} might be collinear to accruals, the link between depreciation and PPE_{it} being quite strong.

In the short-term version of the model we adopt, we omit DEP on the LHS and PPE on the RHS of Equation (5). In this case, Equation (6) obtains:

$$\frac{CA_{it}}{A_{i,t-1}} = \alpha_s \left(\frac{1}{A_{i,t-1}} \right) + \delta_s \left(\frac{\Delta REV_{it}}{A_{i,t-1}} \right) + \varepsilon_{it} \quad (6)$$

Finally, note that, in principle, even though the Jones model is simply an accounting specification of non-discretionary accruals, it remains perfectly compatible with our investment setting (Equation 4), as the explanatory variables appearing in the Jones model may be included in the vector of control variables \mathbf{X} . However, investigating an investment specification, the introduction of these variables in accruals models is not only justified by a statistical bijection between a component of accruals and its accounting determinants, like the accounting dependence of working capital on change in revenues. For instance, in Equation (6), with our investment framework, the change in revenues can be interpreted as a standard liquidity constraint.

2.2.2. The Cash-flows Augmented Jones Model

To account for firm performance it is common to introduce cash-flows (CF) in the accrual models (e.g., Dechow, 1994; McNichols, 2002; Francis *et al.*, 2005 and Zhang, 2007). In its long-term form, this standard accrual model (model II) can be written as:

$$\frac{TA_{it}}{A_{i,t-1}} = \alpha_s \left(\frac{1}{A_{i,t-1}} \right) + \beta_s \left(\frac{PPE_{it}}{A_{i,t-1}} \right) + \delta_s \left(\frac{\Delta REV_{it}}{A_{i,t-1}} \right) + \kappa_s \left(\frac{CF_{it}}{A_{i,t-1}} \right) + \varepsilon_{it} \quad (7)$$

The corresponding short-term version of Equation (7) we consider is then:

$$\frac{CA_{it}}{A_{i,t-1}} = \alpha_s \left(\frac{1}{A_{i,t-1}} \right) + \delta_s \left(\frac{\Delta REV_{it}}{A_{i,t-1}} \right) + \kappa_s \left(\frac{CF_{it}}{A_{i,t-1}} \right) + \varepsilon_{it} \quad (8)$$

In our framework, the introduction of the cash-flows variable may not be viewed simply as an *ad hoc* way of controlling for firm performance, but as a variable proxying for the liquidity and financial constraints stemming from market imperfections. In other words, our approach provides a direct, theoretically founded justification of the influence of the cash-flows variable on firms' accruals.

A standard procedure often found in the accounting literature is to lag cash-flows to correct for endogeneity. This procedure is generally adequate to mitigate the error term autocorrelation but less appropriate to tackle the endogeneity issue *per se* (Theil, 1953). Hence, in our approach, we replace the cash-flows variable by its predicted (fitted) value to ensure its orthogonality with the error term. An intuitive justification for doing so is that accruals are often value related, particularly so for outperforming firms, that

is firms characterized by high Tobin's q and strong persistence in sales. Indeed, for these firms, accruals are strongly (positively) autocorrelated and also quite correlated with cash-flows. Relatedly, even though accrual persistence can be partly attributable to a cosmetic smoothing through the strategic allocation of accruals over few accounting periods, and to various earnings management practices or adjustment costs⁴ (including hiding information on current sales innovation), accrual persistence is also explained by firm performance, and, consequently, by expected cash-flows. As a matter of fact, note that this observation directly relates to the investment perspective on accruals. Indeed, the literature on firm investment suggests that cash flows are one of the main driving forces of investment. Hence, if accruals can be considered as a form of short-term investment, the cash-flows variable should be found a positive, significant factor influencing accruals.

2.2.3. The Tobin's q Accrual Model

Consistent with Zhang (2007) view on the investment perspective of firms' accruals, we propose a new accrual model, model III, for which we adapt Equation (4), and directly introduce Tobin's q as an explanatory variable of firms' accruals:

$$\frac{TA_{it}}{A_{i,t-1}} = \beta_1 \left(\frac{1}{A_{i,t-1}} \right) + \beta_2 \frac{PPE_{it}}{A_{i,t-1}} + \beta_3 \frac{\Delta REV_{it}}{A_{i,t-1}} + \beta_4 \frac{q_{it}}{A_{i,t-1}} + \beta_5 \frac{CF_{it}}{A_{i,t-1}} + \xi_{it} \quad (9)$$

where Tobin's q is proxied empirically by:

$$\frac{\text{Market value of capital} + \text{Accounting value of debts}}{\text{Accounting value of assets}} \quad (10)$$

Note that q is theoretically defined as a marginal concept, while Equation (10) defines it as an average one⁵. The marginal concept is unobservable, so researchers usually define Tobin's q on an average basis. When q is computed this way, Equation (9) establishes a direct link between accruals and the stock market valuation of the firm. In this sense, q directly controls for firm performance in Equation (9). Note also that there are many empirical proxies of Tobin's q in the investment literature, including variables based on working capital. For example, one popular measure used in financial studies proxies Tobin's q with the ratio of the market value of assets to their replacement cost, as originally defined by Tobin (1969). However, since we work in an accounting framework, we rely on an accounting definition of Tobin's q to be more consistent with the literature. In this literature, Tobin's q is usually measured as the market-to-book ratio, i.e., the market value of equity scaled by its book value. However,

⁴ Note that the neoclassical theory of investment suggests that autocorrelation will be present in the estimation of the investment function because of the convex cost of adjusting the stock of capital to its target level.

⁵ Note that marginal and average q are equal if the firm's production function and the investment adjustment cost function are first-degree homogenous and firms operate in competitive markets (Hayashi 1982). Under these assumptions, we might thus expect a close link between the market valuation of a firm and its investment decisions.

given the investment perspective of accruals we investigate, we slightly depart from this common practice to be more consistent with Tobin's q theory, and we consider instead the market value of equity plus the book value of debt, divided by assets. As for the previous models, we primarily focus on the short-term version of Equation (9):

$$\frac{CA_{it}}{A_{i,t-1}} = \beta_1 \left(\frac{1}{A_{i,t-1}} \right) + \beta_3 \frac{\Delta REV_{it}}{A_{i,t-1}} + \beta_4 \frac{q_{it}}{A_{i,t-1}} + \beta_5 \frac{CF_{it}}{A_{i,t-1}} + \xi_{it} \quad (11)$$

As discussed earlier, in the finance literature, Tobin's q is known to be one of the most predominant variables explaining investment. To the extent that accruals decisions can indeed be cast in terms of investment strategy, the analysis of Tobin's q explanatory power seems quite natural. Actually, accrual models already include a return measure, often the return on assets, ROA , to control for the non-linear effect of firm performance (Dechow and Dichev, 2002; Kothari *et al.*, 2005). However, authors omit to directly include Tobin's q , and this can potentially lead to a strong colinearity issue if the return on investment and cash-flows are actually mixed together. This kind of drawback might still apply to Tobin's q as well. Indeed, as the literature suggests, when using the q average measure instead of the theoretical unobservable marginal measure, cash-flows might embed information about Tobin's q , and cash-flows and Tobin's q could be colinear. However, as explained in the following section, this matter can be dealt with by properly accounting for measurement errors (Gilchrist and Himmelberg, 1995; Erickson and Whited, 2000, 2002).

■ 3. Estimation Procedure

Kang and Sivaramakrishnan (1995) and Kang (2005) argue that OLS accrual estimations can deliver misleading results, and the authors advocate instead the use of the IV approach and the GMM to deal with the errors-in-variables, omitted variables and simultaneity problems. Consequently, since accruals and Tobin's q are generally measured with errors, we introduce a tailor-made specification error correction method⁶ and apply it to our three accrual models. To detect specification errors in the accrual models we run two sets of regressions. For example, consider model III (i.e. our Tobin's q augmented model). Following Kothari *et al.* (2005), we first run the OLS regressions using Equations (9) and (11), for long-term and short-term accruals respectively, and then we run the following Haus-C artificial regressions:

$$\frac{TA_{it}}{A_{i,t-1}} = \beta_1^* \left(\frac{1}{A_{i,t-1}} \right) + \beta_2^* \frac{PPE_{it}}{A_{i,t-1}} + \beta_3^* \frac{REV_{it}}{A_{i,t-1}} + \beta_4^* \frac{q_{it}}{A_{i,t-1}} + \beta_5^* \frac{CF_{it}}{A_{i,t-1}} + \sum_{i=1}^5 \varphi_i \hat{w}_{it} + \varepsilon_{it}^* \quad (12)$$

$$\frac{CA_{it}}{A_{i,t-1}} = \beta_1^* \left(\frac{1}{A_{i,t-1}} \right) + \beta_3^* \frac{\Delta REV_{it}}{A_{i,t-1}} + \beta_4^* \frac{q_{it}}{A_{i,t-1}} + \beta_5^* \frac{CF_{it}}{A_{i,t-1}} + \sum_{i=1}^4 \varphi_i \hat{w}_{it} + \varepsilon_{it}^* \quad (13)$$

⁶ For details on this method, see the appendix.

where the \hat{u}_i are the residuals obtained from the regressions of the endogenous variables on the higher moment instrumental variables. In line with Larcker and Rusticus (2010), these higher moment instruments are robust, and have also the advantage of requiring no extraneous information from the models. Equations (12) and (13) represent the generalized version of the augmented accrual model for the long-term and short-term horizons, respectively. Note that the estimated coefficients, φ_i allow the detection of specification errors, and that their signs indicate whether the corresponding variable is overstated or understated in the OLS regression. The β^* estimated in these equations are equivalent to TSLS estimates, but our method offers the key advantage of providing additional information about the severity of the specification errors. Indeed, the ϕ_i measure the bias in the sensitivity of accruals to the i^{th} explanatory variable. If the ϕ_i associated with the i^{th} regressor is significantly positive, then the corresponding β will be lower in the artificial regression (and vice-versa if ϕ_i is negative). In general, we should expect a high positive correlation between $\hat{\beta}_i - \hat{\beta}_i^*$, the estimated error in the coefficient of variable i , and $\hat{\varphi}_i$, the estimated coefficient of the corresponding artificial variable \hat{u}_i . We can sum up the former argument using the following equation:

$$\forall i \quad Spread_i = \pi_0 + \pi_1 \varphi_i + \zeta_i \quad (14)$$

where $Spread_i = \hat{\beta}_i - \hat{\beta}_i^*$. According to Equation (14), the ϕ_i indicate the degree of overstatement or understatement of the OLS estimation, and the goodness of fit of the equation provides information about the severity of the specification errors. This constitutes a straightforward variant of the original Hausman test.

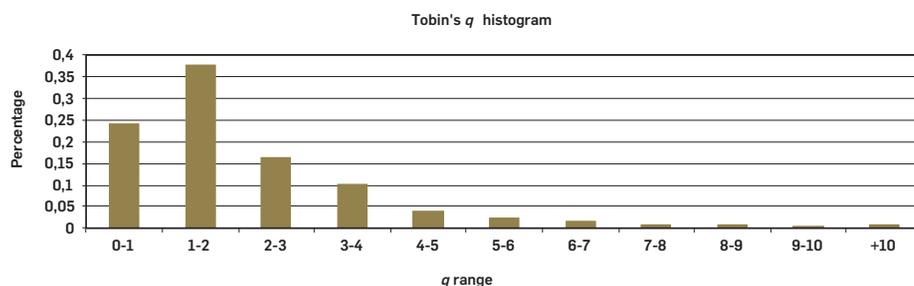
■ 4. Empirical Results

4.1. Data

From a distributional perspective, a deviation of the distribution of earnings from the normal one should indicate earnings management. However, to explain the accrual conundrum (McNichols and Wilson, 1988; Sloan, 1996; Dechow and Dichev, 2002) related to the stationarity of revenues and expenses (Yaari *et al.*, 2007), it is often assumed that the ratio of accruals to earnings is actually a random variable. In other words, abnormal accruals might not only reflect earnings management of discretionary accruals, but also changes in the underlying economic models and firm performance. Relatedly, if earnings management uses forward-looking information, it increases the predictability of accruals. However, it is precisely high-performance firms which present the most persistence in earnings management. High performance may thus erroneously lead the researcher to classify abnormal accruals as discretionary when in fact the residuals of the accrual models still contain information on firm performance. Hence, the challenge is to arrive at specifications which can

disentangle earnings management from firm performance in the residuals. In this respect, the main advantage of Tobin's q is that, by construction, it is particularly well suited to control for firm performance, so that the model should be able to isolate earnings management in the discretionary accruals used to forecast stock returns. To confirm this, we need to analyze data on high performance firms, so we apply our framework to a sample composed of all the non financial firms registered in the S&P500 index. For the exercise, the observations are retrieved from the U.S. COMPUSTAT database, data are annual, and run from December 1989 to December 2006. As previously done by most researchers (e.g., Kothari *et al.*, 2005), we exclude firms displaying missing observations. After having discarded, among the 500 most performing firms constituting the index, the firms with missing information, we have a total of around 10000 pooled observations.

Figure 1. Sample distribution of firms Tobin's q



Since the objective of this study is to shed light on the relationship between accruals and key investment factors, instead of analyzing industrial sectors individually, we need to study representative firms. Figure 1 gives the frequency distribution of Tobin's q in our sample. Given the dispersion of this distribution, it is indeed legitimate to focus on pooled data and we do not have to rely on a complementary sectoral analysis. We adjust firm data for size and run our regressions using pooling methods. Each year the sample does not vary much given that, in our dataset, most firms are good performers and the sample is quite homogeneous, which, *per se*, mitigates the issue of composition effect. Note that this approach is consistent with Ye (2006) who also advocates pooling to improve the goodness of fit of the accrual models. Instead of slicing the sample by year and industry, we thus consider pooling, which offers the additional advantage of a more parsimonious approach for testing the presence of measurement errors.

Total accruals are computed using a balance sheet approach, i.e. change in non-cash current assets (Compustat #4 - #1) less change in current liabilities (Compustat #5), excluding the current portion of long-term debt (Compustat #44), less depreciation (Compustat #14) and taxes (Compustat #71). The annual cash-flows variable is

operating cash-flows computed as the mean value of the monthly data. $\Delta Sales$ is the difference of revenues in year t and revenues in year $(t-1)$ (Compustat #12). PPE , the acronym for property, plant and equipment, is measured at the end of year t (Compustat #7).

4.2. OLS estimations

In Table 1 we provide the OLS estimation results for the long-term accrual models, which serve as a benchmark in this study. Correcting for heteroskedasticity and treating size effects, based on the adjusted R^2 (at 0.74, 0.77 and 0.80 for the three models, respectively) the equations seem to perform quite well. First note that, as conjectured, the best model in terms of adjusted R^2 is our model Tobin's q accrual model. Second, in the equation, except for $\Delta Sales$, all the coefficients are significant at the 99% confidence level. The Durbin-Watson (DW) statistics are quite similar across models, ranging from 1.98 to 2.12 and, in light of the R^2 , there thus seems to be no apparent autocorrelation or non-stationary residuals problems⁷.

● Table 1. OLS estimation, long-term versions

	Model I	Model II	Model III
$1/A_{i,t-1}$	-0.0873	2.7080***	0.3892***
$PPE_{it}/A_{i,t-1}$	-0.1686***	-0.0514***	-0.0554***
$\Delta SALES_{it}/A_{i,t-1}$	0.0026	0.0005	0.0003
$CF_{it}/A_{i,t-1}$		0.3336***	0.3086***
$q_{it}/A_{i,t-1}$			4.0897***
Adjusted R^2	0.74	0.77	0.80
DW	2.01	2.12	1.98

Note. The long-term versions of models I, II and III are described respectively in Equations (5), (7) and (9). TA represents total assets; CF , cash-flows; $\Delta SALES$, the change in sales; ROA , the return on assets; PPE , property, plant and equipment and q , Tobin's q . The explanatory variables are scaled by lagged assets to account for heteroskedasticity. Asterisks indicate the significance levels: *stands for 10%, **stands for 5% and *** stands for 1%.

When comparing the coefficients of models II and III, note the similarities in terms of values and signs of the coefficients. For instance, the coefficient of PPE is -0.0514 in model II, and -0.0554 in model III. We obtain the same results for the estimated coefficients of CF respectively 0.3336 and 0.3086 (and $\Delta SALES$, 0.0005 versus 0.0003). Remark that, *prima facie* the positive sign of CF might appear somewhat surprising. After all, total accruals (not necessarily short-term) are typically high when cash-flows are low, and vice-versa, and, as a result, accruals are negatively correlated with contemporaneous cash-flows. However, total accruals are also positively correlated with

⁷ Note that we may suspect a stationarity problem when the R^2 is high and the DW is low.

lagged and leaded cash-flows (Dechow and Dichev, 2002). Indeed, remind that accruals can be viewed as a smoothed measure of *CF*. Hence, even if the contemporaneous *CF* are negatively correlated with accruals, since we consider the twelve month average of *CF* it is not so surprising to get an overall positive correlation. Besides, this finding is perfectly consistent with the investment literature. In other respects, note that for model I, the estimated coefficient of *PPE* and $\Delta SALES$ are larger, at -0.1686 and 0.0026 respectively. Obviously, this finding is partly attributable to the omission of cash-flows. In fact, the correlation between *PPE* and cash-flows is equal to 0.70 in our sample, which suggests that a great proportion of the impact of *PPE* is transferred to cash-flows when shifting from model I to model II, the coefficient of cash-flows being equal to 0.3336 in model II. Overall, these results suggest that accruals are indeed sensitive to cash-flows, a fact consistent with the investment approach we adopt. The traditional intuition here is that market imperfections and financial constraints influence (short-term) investment, and this shows up in the explanatory power of cash-flows. More importantly, the introduction of Tobin's *q* also delivers results in the same vein. Consistent with the investment theory, to the extent that accruals can be viewed as a form of short-term investment, they must be strongly driven by Tobin's *q*. Our results clearly support this view, as the Tobin's *q* coefficient is equal to 4.0897 and significant at the 99% confidence level.

● **Table 2. OLS estimation, short-term versions**

	Model I	Model II	Model III
$1/A_{i,t-1}$	-0.1868^{***}	-0.2032^{***}	-0.1338^{***}
$\Delta SALES_{it}/A_{i,t-1}$	0.0040^{***}	0.0086^{***}	0.0047
$CF_{it}/A_{i,t-1}$		0.3595^{***}	0.2234^{***}
$q_{it}/A_{i,t-1}$			5.2361^{***}
Adjusted R^2	0.71	0.73	0.74
DW	2.04	1.93	2.23

Note. The short-term versions of models I, II and III are associated respectively with Equations (6), (8) and (11). The definition of the variables is provided in Table 1. The explanatory variables are scaled by lagged assets to account for heteroskedasticity. Asterisks indicate the significance levels: * stands for 10%, ** stands for 5% and *** stands for 1%.

Table 2 reports the results for our short-term models. In spite of the omission of the *PPE* variable, and the associated removal of the depreciation component of accruals, the results remain very comparable to those of the long-term versions, both in terms of sign and magnitude of the coefficients. In particular, they clearly indicate that the variables traditionally used as regressors in investment equations are also significant explanatory variables of firms' accruals, hence supporting the thesis that accruals are indeed a form of short-term investment.

However, there are some differences between the results obtained from the estimation of the short-term versus the long-term versions of our models. Compared to the benchmark models, the impact of $\Delta SALES$ appears more important in the short-run. For instance, in model II, the coefficient of $\Delta SALES$ is respectively 0.0005 (not significant) and 0.0086 (significant) in the long-term and short-term versions. More importantly, note that the influence of the Tobin's q coefficient is also larger in the short-term version (5.2361) compared to the long-term one (4.0897). The greater value of the Tobin's q coefficient is partly attributable to the cash-flows variable, whose coefficient decreases from 0.3086 to 0.2234 when shifting from the long-term to the short-term model. But in any case, the fact that q exerts a stronger influence at short horizon is quite consistent with the short-term investment perspective on firms' accruals.

As a final remark, note that the DW statistics are rarely reported in the accruals studies. Yet, the residuals of the estimated accrual models – i.e., the discretionary accruals – should not be autocorrelated, because if they were, the returns forecast on which they are often based would be biased. Looking at the data, we find that accruals are indeed autoregressive. However, the influence of earnings management cannot last indefinitely and the residuals ought to converge to zero eventually. Dechow and Dichev (2002) regress working capital on lead and lag of cash-flows, which, as noted previously, might constitute an indirect way of controlling for accruals autoregressivity. In our case, we follow Beneish (1997) and Dechow *et al.* (2003) and add autoregressive terms in the regressions going backwards, up to five periods to control for reversals. Using this method to control for the accruals autocorrelation improves the fit of the models, and the DW statistics suggests no remaining autocorrelation in the residuals.

4.3. Haus-C estimations

Tables 3 and 4 present the results of the corresponding Haus-C estimations for the three models corrected for heteroskedasticity. For the three models, the levels of the DW statistics do not seem to indicate any significant autocorrelation. As expected, although most variables are significant at the 95% confidence level, Table 3 indicates that the Haus-C regressions systematically yield lower R^2 , 0.28, 0.48 and 0.39 respectively. This confirms that measurement errors in the explanatory variables indeed cause significant biases in the OLS regressions. More importantly, looking at the significance levels, model III seems clearly to outperform the other models. For instance, as reported in Table 3, the coefficient of $\Delta SALES$ is significant in model III even in the long-run, whereas it is found insignificant in model II. Once again, the estimated impact of PPE on accruals seems overstated in model II relative to model III. Actually, when shifting from model II to model III, the decrease, in absolute value, in the PPE coefficient, from -0.1605 to -0.0786 coincides with an increase of the cash-flows coefficient from 0.1435 to 0.2866. This suggests that the greater influence of PPE in model II is likely due to misspecification.

● Table 3. Haus-C estimations, long-term versions

	Model I	Model II	Model III
$1/A_{i,t-1}$	1.3504***	0.2161***	0.6714***
$PPE_{it}/A_{i,t-1}$	-0.0142***	-0.1605***	-0.0786***
$\Delta SALES_{it}/A_{i,t-1}$	0.0175***	0.0021	0.0165***
$CF_{it}/A_{i,t-1}$		0.1435***	0.2866***
$q_{it}/A_{i,t-1}$			6.3103***
\hat{w}_{1t}	7.9290***	-0.8610*	7.2810***
\hat{w}_{2t}	0.0150	0.1455***	0.0840***
\hat{w}_{3t}	-0.0778***	0.0104***	-0.0612***
\hat{w}_{4t}		-0.1313***	-0.3182***
\hat{w}_{5t}			9.6702***
Adjusted R^2	0.28	0.48	0.39
DW	2.10	1.20	2.11

Note. The long-term versions of models I, II and III are described respectively in Equations (5), (7) and (9). The definition of the variables is provided in Table 1. The explanatory variables are scaled by lagged assets to account for heteroskedasticity. Asterisks indicate the significance levels: *stands for 10%, **stands for 5% and ***stands for 1%. The Haus-C procedure is explained in appendix. There is one artificial Hausman variable, \hat{w}_i , for each explanatory variables of the models (e.g., $i = 1, \dots, 5$ for model III). For instance, \hat{w}_5 is the Hausman artificial variable associated with Tobin's q . It is the residuals of the OLS regression of Tobin's q on the chosen instruments. The coefficient of the variable \hat{w}_5 gauges the measurement error of this variable. A positive sign indicates that the impact of the variable is overstated in the OLS regression, while a negative sign indicates the opposite.

● Table 4. Haus-C estimations, short-term versions

	Model I	Model II	Model III
$1/A_{i,t-1}$	1.6067***	1.4031***	0.4142**
$\Delta SALES_{it}/A_{i,t-1}$	0.0787***	0.0647***	0.0622**
$CF_{it}/A_{i,t-1}$		0.3835***	0.1290*
$q_{it}/A_{i,t-1}$			8.6652***
\hat{w}_{1t}	3.1363***	3.2952***	5.6220***
\hat{w}_{2t}	-0.0749***	-0.0589***	0.0613***
\hat{w}_{3t}		-0.4926	-0.2492***
\hat{w}_{4t}			-11.1660***
Adjusted R^2	0.13	0.18	0.21
DW	2.06	2.20	2.04

Note. The short-term versions of models I, II and III are given respectively by Equations (6), (8) and (11). The definition of the variables is provided by Table 1. The explanatory variables are scaled by lagged assets to account for heteroskedasticity. Asterisks indicate the significance levels: *stands for 10%, **stands for 5% and ***stands for 1%. The Haus-C procedure is explained in appendix. There is one artificial Hausman variable, \hat{w}_i , for each explanatory variables of the models (e.g., $i = 1, \dots, 5$ for model III). For instance, \hat{w}_5 is the Hausman artificial variable associated with Tobin's q . It is the residuals of the OLS regression of Tobin's q on the chosen instruments. The variable \hat{w}_5 gauges the measurement error of this variable. A positive sign for its coefficient indicates that the impact of the variable is overstated in the OLS regression, while a negative sign indicates the opposite.

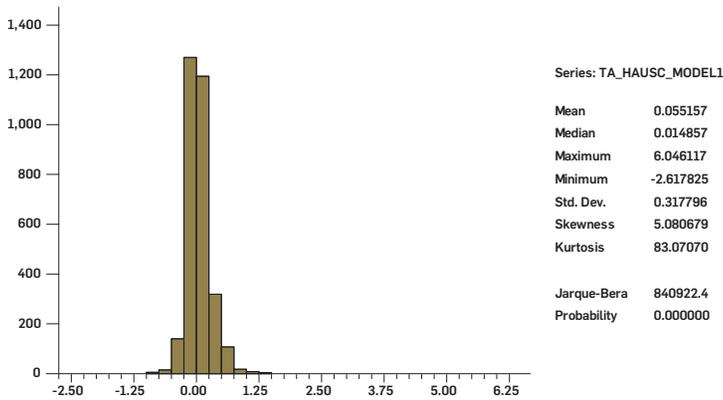
As expected, the $\hat{\phi}_i$ indicate the presence of substantial measurement errors for all the explanatory variables. First, Table 3 reveals that the most commonly used explanatory variables of accruals, $\Delta SALES$, PPE and $1/A_{i,t-1}$, seem to be measured with significant error, which translates into misspecification. One common explanation for this is that these accounting variables are used in accrual models as proxies for economic values. The error on $1/A_{i,t-1}$ is particularly severe, which could explain the great instability of this coefficient and its changing sign, when moving from one specification to another. Second, the coefficient of $\Delta SALES$ changes substantially from one model to another, and it thus seems to be quite contaminated. More precisely, for model III, the $\hat{\phi}_i$ coefficient of $\Delta SALES$ is equal to -0.0612 , significant at the 99% confidence level, whereas in the OLS estimation the coefficient of this variable is almost 0. The Haus-C result thus suggests a severe understatement of this coefficient in the OLS estimation. There is also a significant measurement error of the PPE variable, its $\hat{\phi}_i$ being equal to 0.0840 in model III, significant at the 99% confidence level. In this case, there is an overstatement of the coefficient in the OLS regression.

More importantly, in the long-run, the cash-flow coefficient doubles when Tobin's q is introduced. In model III, the cash-flow coefficient is equal to 0.2866, significant at the 99% confidence level, with a coefficient of understatement of -0.3182 , significant at the 99% level, whereas in model II, from which Tobin's q is absent, the coefficient is lower, at 0.1435, with a coefficient of understatement of -0.1313 . It would be tempting to think that this result is attributable to collinearity. However, the correlation between cash-flows and Tobin's q is close to 0 in our sample. In other words, the introduction of Tobin's q clearly increases the sensitivity of accruals to the other explanatory variables, suggesting that it improves the general fit of the accrual model, especially if errors-in-variables are properly accounted for. Not surprisingly, the Haus-C results confirm the expected positive relationship between accruals and Tobin's q , the influence of this regressor being significant at the 99% confidence level. Consistent with the conventional view that proxies of marginal Tobin's q are usually badly measured, the coefficient of Tobin's q estimated by OLS is about 4.0897 in model III, and much higher, at 6.3103, significant at 95%, when estimated with the Haus-C procedure. The error adjustment variable, \hat{w}_{3t} , at 9.6702, thus confirms the measurement error of this variable.

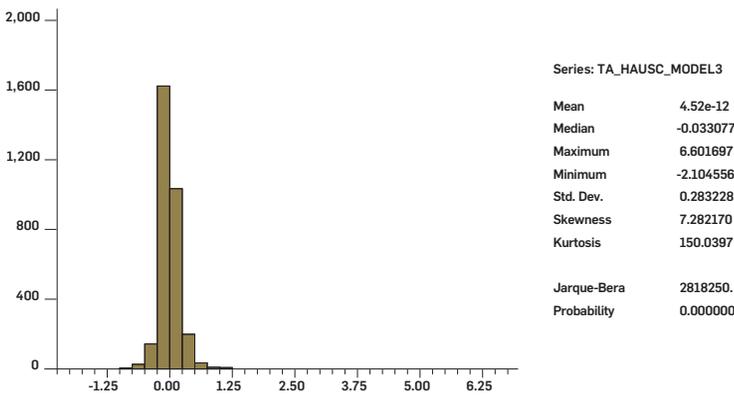
In other respects, as reported in Table 4, the adjusted R^2 is almost halved when we remove the PPE variable. In the short-term version of the models, the levels of all the coefficients are also lower. However, the specification remains qualitatively robust, and the explanatory variables are still significant and of the right sign. More importantly, note that, consistent with the OLS results, when excluding PPE from model III, the influence of Tobin's q is strengthened, and the impact of cash-flows is divided by two, its coefficient being only significant at the 90% confidence level. Not only is the influence of Tobin's q relative to cash-flow higher in the short-term model

III, but it is even larger if we account for measurement errors. Going from OLS to Haus-C, the coefficient increases from 5.2361 to 8.6652. This result could be paralleled to the one reported by Erickson and Whited (2000) for (long-term) investment. Quite counter-intuitively however, everything works as if the conventional theory of investment applied more at shorter horizons, firm performance influence on short-term investment and earnings management being reinforced, whilst cash-flows influence would be dampened. To understand this paradoxical finding, we have to bear in mind the fact that in the short-term accrual models, *PPE* is excluded. Since this variable, as often documented in previous studies, is highly correlated with cash-flows, it should not be too surprising to find cash-flows more significant in the long-term version of the models, including *PPE*, and this, regardless of the way errors-in-variables are treated. In other words, the lack of significance of cash-flows in the short-term accrual models is partly an artefact of the correlation between *PPE* and cash-flows.

Figure 2. Total discretionary accruals estimated with the Haus-C method, model I

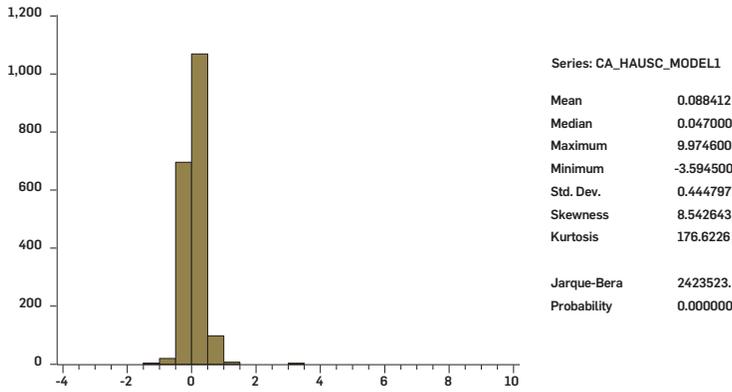


Total discretionary accruals estimated with the Haus-C method, model III

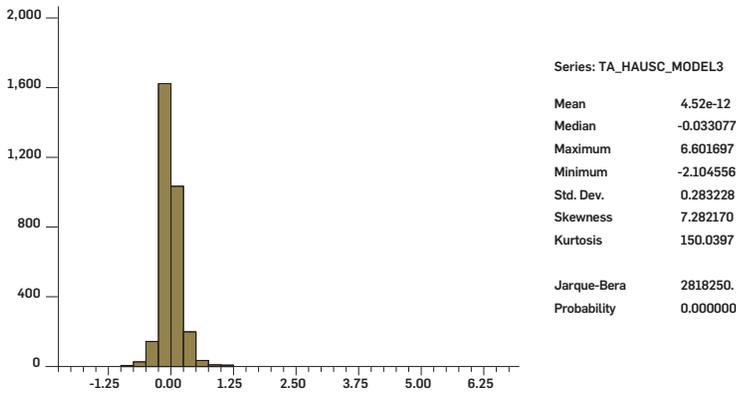


Note. These histograms are built using the residuals of the accrual (long-term) models I and III whose estimation appears in Table 3, discretionary accruals being the residuals of the estimated accrual models.

■ **Figure 3. Current discretionary accruals estimated with the Haus-C method, model I**



Current discretionary accruals estimated with the Haus-C method, model III



Note. These histograms are built using the residuals of the accrual (short-term) models I and III, whose estimation appears in Table 4, discretionary accruals being the residuals of the estimated accrual models.

■ 5. Accruals Residuals Analysis

The problem with the residuals of the accrual models used to forecast stock returns is that discretionary accruals do not necessarily reflect earnings management only, since accruals are also related to firm performance. Consequently, differences in estimated discretionary accruals can be due to performance characteristics rather than incentives to manage earnings — particularly so if the relationship between accruals and performance is nonlinear. Given the significant role played by firm performance, our motivation to consider accruals as a form of investment appears quite natural. In this respect, the main contribution of this study is to show that Tobin's q , a key explanatory variable of firm investment, indeed strongly influences firms' accruals. In this paper, we rely on OLS but also Haus-C to estimate our Tobin's q augmented accrual model,

arguing that it delivers a robust fit of firms' accruals. Logically, we should then expect that this type of specification is also able to deliver residuals which can isolate the earnings management component of discretionary accruals. As a robustness check, it is thus instructive to study the residuals of our regressions – i.e., the discretionary part of accruals.

From an econometric perspective, the mean of the residuals of a regression ought to be equal to 0 in order to avoid any bias in the estimation. In this respect, *TDA*, total discretionary accruals, also ought to be 0 in the long-run since no earnings management practice can influence financial results indefinitely (Ronen and Yaari, 2008). Figure 2 and Figure 3 provide the distributions of total discretionary accruals, *TDA*, and current discretionary accruals, *CDA*, respectively, both expressed in terms of total assets for models I and III. Compared to model I, model III seems to perform better along this dimension. Indeed, in Figure 2, note that the mean of *TDA* is equal to 0.055⁸ when estimated with model I, whereas it is practically 0 when estimated with model III. Therefore, having a mean of zero, the *TDA* associated with model III seems appropriate to forecast returns. Relatedly, regardless of the model considered, the *TDA* distribution seems positively skewed. For instance, for model III, the skewness coefficient is equal to 7.28, which supports the conventional view that discretionary accruals are likely influenced by various earnings management practices. As a matter of fact, it is remarkable to see that, while the mean of model III residuals is lower, the skewness of the residuals is actually higher. One obvious explanation is that, by effectively controlling for firm performance, Tobin's *q* is indeed able to isolate the earnings management information contained in discretionary accruals.

In other respects, as it is the case for *TDA*, the *CDA* mean goes down to 0 when the investment variables (cash flows and Tobin's *q*) are introduced (Figure 3). As shown in Figure 3, the *CDA* mean for model I, at 0.088, is higher than the corresponding mean of *TDA*, and the *CDA* skewness coefficient, at 10.47, is much higher than its *TDA* counterpart. This confirms that discretionary accruals are indeed manipulated, especially at short horizons⁹. A look at the skewness coefficient supports this view, as the larger coefficient observed for model III (10.47 versus 8.54 for model I) suggests that the distribution of the residuals can no longer be attributable to firm performance. Overall, our results suggest that the residuals of model III are well suited for financial analysis because they appear purged from the systematic link between stock returns and investment.

⁸ Remind that accruals are defined in terms of assets.

⁹ Accruals arise from a discrepancy of timing between cash-flows and the accounting recognition of the transaction. In this respect, accruals management is thus a short-term phenomenon which vanishes in the long-run. Over the firm's lifetime, reported revenues must equal total cash inflows, and total accruals must equal zero (Ronen and Yaari, 2008).

■ 6. Conclusion

Some econometric challenges are related to the estimation of our augmented accrual model. A well-known issue relates to the difficulty of properly estimating Tobin's q given that it is not directly observable. As usually done in the investment literature, we rely on an average measure to proxy Tobin's q . There is thus an inherent measurement error related to the computation of this ratio. Ignoring the appropriate correction would entail the usual empirical interaction between cash-flows and Tobin's q , biasing the estimated coefficients of both variables. We thus resort to a specific estimation procedure to tackle this measurement error. Based on this methodology, we are also able to detect serious measurement errors in the basic accrual model and its augmented versions. Our findings reveal that the differences between the coefficients obtained from the IV method and those resulting from the standard OLS are actually quite substantial, which suggests the presence of significant measurement errors in all variables.

More importantly, our main contribution is to show that Tobin's q is a significant explanatory variable of firms' accruals. Consistent with the literature on firm investment, our results support the idea that financial constraints also influence accruals management. However, we find that the impact of cash-flows is actually *reduced* when simultaneously introducing Tobin's q in the short-term version of the model. Clearly, this particular phenomenon is detectable with the IV estimation method we introduce.

Despite its merits, our study leaves many questions open to investigation. Accrual models aim at analyzing the fundamental factors which normally influence cash-flows smoothing in order to identify earnings management patterns with the residuals – i.e., with the discretionary accruals. With the introduction of Tobin's q , we can derive “appropriate” estimated residuals, in the sense that they appear quite close to zero on average. This is due to the fact that, given its theoretical property, Tobin's q effectively removes from accruals residuals the information related to firms' performance. This might prove particularly useful for portfolio managers and financial analysts, since discretionary accruals provide an important information to forecast stocks returns. However, compared to the alternative specifications provided in the literature (Wu *et al.* 2007, 2010) we do not know *a priori* whether the residuals of the Tobin's q augmented model we propose are better suited to forecast stock returns. In this study, our focus is to show that accruals can indeed be cast in terms of short-term investment. Given the positive results we obtain, it will be interesting to investigate the relative performance of our model *vis-à-vis* competing ones (e.g., Beneish, 1997) to forecast stock market returns. Considering the nonlinear relationship between accruals and firm performance, the autoregressive behavior of accruals, and the skewness coefficients we obtain, it would be interesting to address this question using a GARCH framework. This is left for future work.

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■ Appendix A: The Choice of Instruments¹⁰

Since accrual models present specification errors the condition of orthogonality is generally violated and the estimators of the coefficients of the models are not unbiased and consistent. To reduce the estimation biases in these coefficients, we thus regress, in a first pass, the endogenous explanatory variables on instruments. The delicate part is to judiciously choose these instruments. To deal with specification errors, Geary (1942), Durbin (1954), Kendall and Stewart (1963), Pal (1980), Fuller (1987), and more recently Dagenais and Dagenais (1997), Lewbel (1997) and Meng *et al.* (2011) have proposed instruments based on higher moments and cumulants. Racicot and Théoret (2012) generalize these instruments and apply them to financial models of returns, testing and correcting specification errors in a GMM framework.

The set of new instruments we propose to build an estimator accounting for specification errors (and more specifically measurement errors) is based on an *optimal* combination of the estimators of Durbin (1954) and Pal (1980). Let us first assume the following general form $y = \alpha + \mathbf{X}\beta$, where y is the vector ($n \times 1$) representing the dependent variable, here accruals, and \mathbf{X} is the matrix ($n \times k$) of the explanatory variables¹¹. β is the ($k \times 1$) vector of parameters to estimate. Assume also the existence of specification errors in the explanatory variables which might create inconsistency in the estimation of the β vector. To tackle this issue, Durbin (1954) proposes to use as instruments the following product: x^*x , where x is the \mathbf{X} matrix of the explanatory variables expressed in deviation from the mean, and where the symbol $*$ stands for the Hadamard element by element matrix multiplication operator. In the same vein, Pal (1980) introduces as instruments cumulants based on the third power of x instead of the squares. Combining these instruments, we obtain a new matrix of instruments \mathbf{Z} based on the cumulants and co-cumulants of \mathbf{x} and \mathbf{y} , these being the matrix \mathbf{X} and the vector \mathbf{y} expressed in deviation from the mean. This \mathbf{Z} matrix may be partitioned into k vectors or series, i.e. $\mathbf{Z} = [z_1 \ z_2 \ \dots \ z_k]$. The vector z_1 , built with the first explanatory variable, is the instrument of the first explanatory variable, and so on. We regress the explanatory variables on this vector \mathbf{Z} to obtain \hat{x} :

$$\hat{\mathbf{x}} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1} \mathbf{Z}'\mathbf{x} \quad (15)$$

Then the new optimal instruments $\hat{\mathbf{w}}^c$ based on cumulants of the explanatory variables are defined as:

$$\hat{\mathbf{w}}^c = \mathbf{x} - \hat{\mathbf{x}} = [\hat{w}_1^c \ \hat{w}_2^c \ \dots \ \hat{w}_k^c] \quad (16)$$

¹⁰ See: Racicot and Théoret (2012).

¹¹ In model III of short-term accruals the matrix \mathbf{X} is equal to $[1/A_i \ \Delta Sales_i/A_i \ CF_i \ q_i/A_i]$.

In their study, Racicot and Théoret (2012) find that this kind of instruments appear orthogonal to the estimated residuals. The correlation between \hat{w}_i^c and the corresponding explanatory variable x_i is around 90%, the correlation is close to 0 with the other explanatory variables, and in this sense, these instruments can be considered optimal. To improve the existing instrumental methods used to tackle the endogeneity issue in accrual models, we adopt the \hat{w}_i^c instruments they developed with a modified version of the Hausman (1978) artificial regression.

■ Appendix B: The Augmented Hausman Artificial Regression¹²

To detect specification errors in our sample of firms, we could use the original Hausman h test¹³ with the following classical linear regression model: $y = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where \mathbf{y} is a $(n \times 1)$ vector representing the dependent variable; \mathbf{X} , a $(n \times k)$ matrix of the explanatory variables; $\boldsymbol{\beta}$, a $(k \times 1)$ parameters vector, and $\boldsymbol{\varepsilon} \sim iid(0, \sigma^2)$. The Hausman test compares two estimates of the parameters vector, $\boldsymbol{\beta}_{OLS}$, the least-squares estimator (OLS), and $\boldsymbol{\beta}_A$, an alternative estimator taking a variety of specifications (the instrumental variables estimator $\boldsymbol{\beta}_{IV}$ in our case). The hypothesis H_0 is the absence of specification errors, and H_1 , their presence. First, note that the vector of estimates $\boldsymbol{\beta}_{IV}$ is consistent under both H_0 and H_1 , whereas $\boldsymbol{\beta}_{OLS}$ is only consistent under H_0 and not consistent under H_1 . Consequently, under H_0 , $\boldsymbol{\beta}_{IV}$ is less efficient than $\boldsymbol{\beta}_{OLS}$. Second, the Hausman test aims at verifying if “the endogeneity” of some variables – in our case the variables measured with errors – has any significant effect on the estimation of the parameters vector. Therefore, the Hausman test is an *orthogonality* test, that is, helping verify if *plim* $(1/T) \mathbf{X}'\boldsymbol{\varepsilon} = 0$ in large samples. To implement the test, researchers then define the following vector of contrasts or distances: $\boldsymbol{\beta}_{IV} - \boldsymbol{\beta}_{OLS}$. The resulting h test statistic reads: $h = (\hat{\boldsymbol{\beta}}_{IV} - \hat{\boldsymbol{\beta}}_{OLS})' [\mathbf{Var}(\hat{\boldsymbol{\beta}}_{IV}) - \mathbf{Var}(\hat{\boldsymbol{\beta}}_{OLS})]^{-1} (\hat{\boldsymbol{\beta}}_{IV} - \hat{\boldsymbol{\beta}}_{OLS}) \sim \chi^2(g)$, with $\mathbf{Var}(\hat{\boldsymbol{\beta}}_{IV})$ and $\mathbf{Var}(\hat{\boldsymbol{\beta}}_{OLS})$ the respective estimates of the covariance matrices of $\hat{\boldsymbol{\beta}}_{IV}$ and $\hat{\boldsymbol{\beta}}_{OLS}$, and g the number of potentially endogenous regressors. H_0 is rejected if the *p-value* of this test is less than α , the critical threshold of the test (e.g. 5%).

Third, and more importantly, note that, according to MacKinnon (1992), the h test might also run into difficulties if the matrix $[\mathbf{Var}(\hat{\boldsymbol{\beta}}_{IV}) - \mathbf{Var}(\hat{\boldsymbol{\beta}}_{OLS})]$, which weights the vector of contrasts, is not positive definite. Since this is the case with most of the accrual models we study, we rely instead on an alternative method to run our Hausman test. For example, assume a five variable linear regression model (e.g., the long-term version of the accrual model incorporating Tobin’s q and cash-flows model III):

¹² For previous applications of this method see Coën and Racicot (2007) and Racicot and Théoret (2012).

¹³ For details on the Hausman test, see: Hausman (1978), Wu (1973), MacKinnon (1992) and Pindyck and Rubinfeld (1998). A very good presentation of the version of the Hausman test using an artificial regression in the context of correction of errors-in-variables may be found in Pindyck and Rubinfeld (1998). They present the case of one explanatory variable, whereas we apply it to the case of multiple explanatory variables.

$$y_t = \beta_0 + \sum_{i=1}^5 \beta_i x_{it}^* + \varepsilon_t \quad (17)$$

with $\varepsilon \sim iid(0, \sigma^2)$.

and that the variables x_{it}^* ¹⁴ are measured with errors, that is:

$$x_{it}^* = x_{it} + U_{it} \quad (18)$$

with x_{it} the corresponding observed variables measured with errors. By substituting Equation (18) in Equation (17), we have:

$$y_t = \beta_0 + \sum_{i=1}^5 \beta_i x_{it} + \varepsilon_t^* \quad (19)$$

with $\varepsilon_t^* = \varepsilon_t - \sum_{i=1}^5 \beta_i U_{it}$. As explained before, estimating the coefficients of Equation (19) by the OLS method leads to biased and inconsistent coefficients because the explanatory variables are correlated with the innovation. Consistent estimators can be found if we can identify an instrument vector \mathbf{z}_t which is correlated with every explanatory variable but not with the innovation of Equation (19). Then we regress the five explanatory variables on \mathbf{z}_t . We have:

$$x_{it} = \hat{x}_{it} + \hat{u}_{it} = \hat{\gamma}_i \mathbf{z}_t + \hat{u}_{it} \quad (20)$$

where \hat{x}_{it} is the value of x_{it} estimated with the vector of instruments, and \hat{u}_{it} the residuals of the regression of x_{it} on \hat{x}_{it} . Substituting Equation (20) into Equation (19), the following artificial regression obtains:

$$y_t = \beta_0 + \sum_{i=1}^5 \beta_i \hat{x}_{it} + \sum_{i=1}^5 \beta_i \hat{u}_{it} + \varepsilon_t^* \quad (21)$$

The explanatory variables of this equation are, on the one hand, the estimated values of x_{it} , obtained by regressing the five variables on the vector of instruments \mathbf{z}_t , and, on the other hand, the respective residuals of these regressions. Therefore, Equation (21) is an augmented version of Equation (19).

We can show that:

$$p \lim \left[\frac{\sum \hat{u}_{it} + \varepsilon_t^*}{N} \right] = p \lim \left[\frac{-\beta_1 \sum x_{it} U_{it}}{N} \right] = -\beta_1 \sigma_{U_i}^2 \quad (22)$$

If there is no specification error, $\sigma_{U_i}^2 = 0$, the OLS estimation results in a consistent estimator for β_1 , the parameter of \hat{u}_{it} in Equation (21), and the coefficient is then

¹⁴ We use the asterisks to designate the unobserved variables.

equal to the one of the corresponding explanatory variable. In the case of specification errors, $\sigma_{\hat{u}_i}^2 \neq 0$ and, therefore, the estimator is not consistent. For detecting the presence of specification errors, as we do not know *a priori* if there are such errors, we first have to replace the coefficients of the \hat{u}_i in Equation (20) by θ_i . We thus have:

$$y_i = \beta_0 + \sum_{i=1}^5 \beta_i \hat{x}_{it} + \sum_{i=1}^5 \theta_i \hat{u}_{it} + \varepsilon_i^* \quad (23)$$

Since according to Equation (20), $\hat{x}_{it} = x_{it} - \hat{u}_{it}$, we can then rewrite Equation (23) as:

$$y_i = \beta_0 + \sum_{i=1}^5 \beta_i x_{it} + \sum_{i=1}^5 (\theta_i - \beta_i) \hat{u}_{it} + \varepsilon_i^* \quad (24)$$

If there is no specification error for x_{it} , then $\theta_i = \beta_i$. In the opposite case, $\theta_i \neq \beta_i$, and the coefficients of the residuals terms \hat{u}_{it} are significantly different from 0. A significantly positive estimate of $(\theta_i - \beta_i)$ indicates that the estimated coefficient of the corresponding explanatory variable, x_{it} , is overstated in the OLS regression. In this case, the estimated coefficient for this variable is lower compared to the OLS one in Equation (24). On the other hand, if the estimated coefficient $(\theta_i - \beta_i)$ is significantly negative, it suggests that the estimated coefficient of the corresponding explanatory variable, x_{it} , is understated by OLS, and consequently the estimated coefficient for this variable is higher in Equation (24). In other respects, the estimated coefficients β_i are identical to those produced by a TSLS procedure with the same instruments (Spencer and Berk, 1981), except that, compared to a strict TSLS, Equation (24) also provides additional information which proves quite helpful when estimating accruals. In the procedure we propose to test for specification errors, we first regress the observed explanatory variables x_{it} on the instruments vector to obtain the residuals \hat{u}_{it} . Then, we regress y_i on the observed explanatory variables x_{it} and on these residuals \hat{u}_{it} . This is the auxiliary (or artificial) regression we just described. If the coefficient of the residuals associated with an explanatory variable is significantly different from 0, we can directly infer the presence of a specification error. In this case, a *t* test is used to assess the *severity* of the specification error. To our knowledge, such a test has never been used in this context. Usually, a Wald test (*F* test) is performed to check whether the whole set of $(\theta_i - \beta_i)$ coefficients is significantly different from zero, but this ignores the case of specification errors associated with a *specific* subset of explanatory variables. We can generalize the former procedure to the case of *k* explanatory variables with our modified Hausman regression. Let \mathbf{X} be a $(n \times k)$ matrix of explanatory variables not orthogonal to the innovation, and let \mathbf{Z} be a $(n \times s)$ matrix of instruments ($s > k$). We regress \mathbf{X} on \mathbf{Z} to obtain $\hat{\mathbf{X}}$:

$$\hat{\mathbf{X}} = \mathbf{Z}\hat{\boldsymbol{\theta}} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1} \mathbf{Z}'\mathbf{X} = \mathbf{P}_z\mathbf{X} \quad (25)$$

where \mathbf{P}_z is the “predicted value maker”. Having run this regression, we can compute the matrix of residuals $\hat{\boldsymbol{w}}$:

$$\hat{\mathbf{w}} = \mathbf{X} - \hat{\mathbf{X}} = \mathbf{X} - \mathbf{P}_z \mathbf{X} = (\mathbf{I} - \mathbf{P}_z) \mathbf{X} \quad (26)$$

and perform the following artificial regression:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \hat{\mathbf{w}}\boldsymbol{\lambda} \quad (27)$$

A F test on the $\boldsymbol{\lambda}$ coefficients indicates whether the $\hat{\mathbf{w}}$ are significant as a group, but we also introduce a t test on each individual coefficient to check whether the corresponding $\boldsymbol{\beta}$ is understated or overstated. The vector of $\boldsymbol{\beta}$ estimated in Equation (27) is identical to the TSLS estimates, that is:

$$\boldsymbol{\beta} = \boldsymbol{\beta}_{IV} = (\mathbf{X}'\mathbf{P}_z\mathbf{X})^{-1} \mathbf{X}'\mathbf{P}_z\mathbf{y} \quad (28)$$

