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Fiscal Policy, Public Expenditure Composition and Growth. Theory and Empirics

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Abstract

This paper responds to the development policy debate involving the World Bank and the IMF on the use of fiscal policy not only for economic stabilization but also to promote economic growth and increase per capita income. A key issue in this debate relates to the effect of the composition of public expenditure on economic growth. Policy makers and some researchers have argued that expenditure on growth-enhancing functions could enhance future revenue and justify the provision of "fiscal space" in the budget. But there are no simple ways to identify the growth-maximizing composition of public expenditure. The current paper lays out a research strategy to explore the effects of fiscal policy, including the composition of public expenditure, on economic growth, using a time series approach.

Based on the modeling strategy of Greiner, Semmler and Gong (2005) we develop a general model that features a government that undertakes public expenditure on (a) education and health facilities which enhance human capital, (b) public infrastructure such as roads and bridges necessary for market activity, (c) public administration to support government functions, (d) transfers and public consumption facilities, and (e) debt service. The proposed model is numerically solved, calibrated and the impact of the composition of public expenditure on the long-run per capita income explored for low-, lower-middle- and upper-middle-income countries. Policy implications and practical policy rules are spelled out, the extension to an estimable model indicated, a debt sustainability test proposed, and the out-of-steady-state dynamics studied.

Keywords: Infrastructure investment, Economic growth, Compostion of the public budget, Financing public expenditures, Public deficit, Public debt.

JEL classification: H1, H2, H3, H4, H5, E2, E6.

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1. Introduction

Starting from a "production function" view of the growth and development process with a narrow focus on physical capital, labor force, and exogenous technical progress, growth theory has come to acknowledge the role of an economy's endowment and continuing investment in human capital, physical infrastructure, research and knowledge development, institutions, and innovation in explaining its growth trajectory.

The implications for policy design are somewhat harder to divine. From the perspective of a government, which policies enhance an economy's growth prospects and how large are policy effects on growth? In part, the answer depends on the empirical approach taken. Studies based on cross-country regressions have explored a large number of forces of growth, but they face methodological perils such as the huge heterogeneity of countries (different technologies, institutions, and preferences across countries), and the uncertainty of presumed underlying models and parameters. Consequently, nonrobustness of the outcomes and ambiguous policy implications are a common criticism of studies based on cross-country regressions[1].

Greiner, Semmler and Gong (2005) argue that a time series perspective on economic growth may be more useful to pursue in designing growth and development strategies. Growth models, as advocated by Greiner and colleagues, (1) allow for better specification of microeconomic behavior of economic agents, (2) enable time series study of the forces of growth for a country or a group of countries at particular stages of economic growth, (3) permit analysis using econometric time series methods, and (4) allow us to spell out (though mostly in the context of small-scale models) important implications for growth and development policies.

Since such models, which lend themselves to some time series tests, are often difficult to treat when a larger number of forces of growth are introduced, Greiner et al. (2005) have suggested identifying the specific forces that are important in a particular country consistent with its particular stage of development. They mainly consider externalities and learning from others, education and human capital, the creation and accumulation of knowledge and public infrastructure as forces of economic growth in a time series context. Overall, this approach allows analysts to better derive the public policy implications for economic growth.

The primary interest of this paper is to explore, as the World Bank's papers to the Development Committee (2006, 2007) suggest, whether countries could better use fiscal

[1] See Rodrik (2005) for a methodological criticism of cross-country regressions in identifying growth effects of policies. See also Easterly (2005) in the Handbook of Economic Growth for a skeptical review of the role of policies in explaining cross-country growth.

policy (and in particular, the level and composition of public expenditure) to promote sustainable increases in growth and welfare for low- and middle-income countries. We are encouraged by some recent work by Glomm and Rioja (2006), Agenor and Neanidis (2006), and country studies undertaken by World Bank research[2].

Section 2 provides a selective discussion of recent literature and empirical evidence on public investment in infrastructure, health, and education and their effect on growth. In section 3 we sketch a general social planner type model that includes choices regarding the accumulation of physical capital as well as the composition of investment in infrastructure, health, and education. In section 4, the model is calibrated by matching actual differences in per capita income for three groups of countries (for low-income, lower-middle-income and upper-middle-income countries). In addition the effects of a change in foreign aid and other factors that have an impact on growth are explored.

Section 5 undertakes a similar calibration exercise; here, however, by considering the growth and welfare effects arising from a change in the composition of the public expenditure. Section 6 discusses sustainability tests of fiscal policy. Section 7 concludes the paper and spells out some practical policy rules. In the appendix the data sources are discussed and the detailed results for the lower-middle-income and upper-middle-income countries are reported. Also, in the appendix the out-of-steady-state dynamics of a simplified version of the model are explored, and it is shown how that the model can be turned into an estimable model using time series data.

2. Literature and Evidence

The scope for policy to influence economic growth depends on the underlying model of growth. So long as the Solow model dominated economists' view of growth, there was little role for fiscal policy to influence the long term rate of growth, which depended on exogenous technical progress[3]. Lucas (1988) and Barro (1990) opened the door to a rich literature on endogenous growth theory and a corresponding attempt to develop our understanding of the implications for fiscal policy. Tanzi and Zee (1997) provide a relatively early review of the resulting literature on fiscal policy and concluded that despite the lack of robust empirical results, endogenous growth theory provided the basis for confidence that fiscal policy could affect long run growth performance of countries.

2. See the papers by Arestoff and Hurlin (2006), Balonos (2005), Ferreira and Arajo (2006), and Suescun (2005).

3. Temple (2003) argues that the scope for policy to have an influence on the level of output should merit the attention of policy makers and analysts but has been neglected because of a misguided focus on effects on the long term growth rate and an undervaluation of level effects.

The theoretical literature on fiscal policy has studied the effect of 'productive' and 'unproductive' spending and distortionary and non-distortionary taxation on long term growth. This literature generally predicts that productive spending financed by non-distortionary taxes will have a positive effect on long term growth whereas the opposite combination (unproductive spending financed by distortionary taxation) will have a negative effect. The algebraic sign of other spend-tax combinations is more ambiguous since there are opposing effects at work. Barro (1990) suggests that an inverted U-curve relationship would exist between productive government expenditure and economic growth, as the rising cost of distortionary taxation necessary to finance spending overtakes the declining benefits of productive spending.

Early models of growth that featured government expenditure used fairly simple characterizations of productive and unproductive spending-public investment was viewed to be productive whereas public consumption was unproductive. Devarajan et al. (1996) developed a model with public investment and consumption expenditure to show that the growth impact of public investment could be negative if there was excessive investment. Glomm and Ravikumar (1997) considered the implications of government expenditure on infrastructure (which influences private production) as well as on education which results in human capital accumulation. More recent literature (Zagler and Durnecker, 2003, Glomm and Rioja, 2006, Blankenau and Simpson, 2004, Agenor and Neanidis, 2006) provide a more disaggregated discussion of government expenditure, typically including spending on public infrastructure, health, and education, which are described as providing inputs for private production. Zagler and Durnecker (2003) define an economy where output is produced using labor, private capital and public infrastructure expenditure and consider the effects of government spending and taxation on long term growth rates. Glomm and Rioja (2006) consider the implications of shifting expenditure from transfers to infrastructure for education and conclude, based on empirical evidence from Brazil, that at the margin the growth implications are small. Blankenau and Simpson (2004) focus on education expenditure and growth.

Some of the papers take account of the interdependence among these expenditures, with the productivity of health spending depending on education and infrastructure expenditure or stocks, and vice versa. The intuition behind such complementarities is well known - good sanitation and water supply infrastructure has large health benefits, including a reduction in incidence of malaria and gastro-intestinal diseases. This in turn has a positive effect on school attendance rates and on learning outcomes (see Bundy and et al., 2005) as well as on labor productivity in market activities.

While theoretical models have become more nuanced about public expenditure and their financing, the empirical evidence is often perceived to be ambiguous, reflecting problems of data, differing research methodologies and econometric techniques. Gemmell (2007) provides a useful review of the evidence, and concludes that more recent literature uses more reliable methods (including a clearer specification of the government budget constraint) to derive robust evidence, at least for OECD countries, of long run impacts of fiscal policy on economic growth. Even for developing countries he finds that, consistent with theory, recent studies show a positive medium to long-run growth effect of certain categories of expenditure, such as transport and communication infrastructure, education and health. However, the complementarities between health, education, infrastructure and growth involve trade-offs in the actual development process. Public resources are limited and given the constraints governments often need to weigh the benefits of expenditure on one against the benefits of spending on the other. In the following section a model is formulated to represent both the complementarities and the trade-offs of public investment.

3. A Model of Fiscal Policy and Economic Growth

The model to be presented is based on the work by Greiner et al. (2005) but takes into account some of the generalizations that have been put forward by Corsetti and Roubini (1996) and others. The current paper develops a model of growth with a private and a public sector, the latter described by a government that can choose to raise resources through taxation, borrowing, or foreign aid. In our model we assume that the tax rate is chosen optimally and there is a well-defined inter-temporal budget constraint. We allow for foreign debt and spell out fiscal policy rules that ensure sustainability of debt. Besides the accumulation of physical capital by the private sector, the model includes public investment in infrastructure, health and education. In contrast to Agenor and Yilmaz (2006) who work with expenditure flows, we develop a model with stocks: private physical capital, human capital and a broad notion of public capital that includes public infrastructure to support market production as well as facilities for health and education services. The paper attempts to derive some policy insights using empirical techniques to estimate the model. Given that time series models that study forces of growth in a historical context are rather complex and require high quality time series data to be estimated, we adopt a calibration technique to assess the growth impact of fiscal policy on countries identified by levels of income/development.

3.1 The Model

The Economy

Production in this stylized economy is characterized by a Cobb Douglas function ($Ak^{\alpha}\,(u_1 h)^{\beta}\,(v_1 g)^{\gamma}$ for the production of market goods and $((1-u_1)h)^{\varepsilon_1}\,(v_2 g)^{\varepsilon_2}\,(v_3 g)^{\varepsilon_3}$ for the production of human capital) with private capital k, public capital g, and human capital h. Note that only a part of public capital ($v_1 g$) is used to support private market production, while other parts are used to help to build human capital in the form of schools ($v_2 g$) and medical centers ($v_3 g$) and do not directly influence production[4]. This realistically describes the longer gestation lag in creating human capital relative to typical physical infrastructure.

The Government

Government (represented by a social planner) raises taxes optimally without distortion and can choose four types of public expenditure: (i) public investment to enhance education and health services that increase the stock of human capital, (ii) investment on public infrastructure assets (transport and communication system as well as energy, water supply and sanitation), to support market production and the creation of human capital, (iii) transfers and public consumption representing expenditure with public goods' characteristics which may enter into households' preferences (public parks, civic facilities and consumption transfers), and (iv) public administration necessary for the functioning of the government (including justice, security and tax collection). These four expenditure streams allow us to consider the effect of public expenditure composition on growth.

The stylized model developed in this paper focuses on deriving the implications for growth and welfare due to alternative government choices regarding the allocation of the stock and flow of public capital. The government can choose to allocate public investment in infrastructure assets (such as roads) that directly influence market production or in assets relevant for human capital accumulation (such as schools and clinics) that may have a lagged effect on growth. The growth and welfare effects are worth studying since they help us highlight the inter-temporal consequences and trade-offs of policy choices confronting many governments[5].

The model enables government to acquire foreign debt and recognizes debt service as a part of the government budget. Foreign transfers to low-income countries are assumed to reflect foreign aid that is earmarked for public investment. The social planner is assumed to maximize a logarithmic welfare function, a form that simplifies preferences.

When we define the variables, all variables are in per capita form and we define public capital as non-excludable but subject to congestion in the production of market

4. *Since the v's represent shares, $v_1 + v_2 + v_3 = 1$.*

5. *To focus on the effects of public capital allocation, the model assumes a stylized world where the government decides on the allocation of public capital and the private sector provides the labor to staff the schools and clinics as well as the labor employed in market production. One could imagine an economy where private sector volunteers provide the human capital services but rely on complementary government facilities. Human capital allocation therefore has no direct impact on the government budget. An extension to the current paper could consider the implication of government employment of teachers and doctors and then consider the compositional choice regarding capital and recurrent wage expenditure.*

goods and the creation of human capital, because it is per capita public capital ($g = G/L$) which affects per capita output.

Table 1. Variable Overview (all in per capita terms)

Variables	Description
b	Debt
c	Consumption
d	Debt service
e_p	Amount of resources absorbed by the public sector
k	Private capital
g	Public capital
h	Human capital
y	GDP

Table 2. Parameter Overview (all in per capita terms)

Parameters	Description
A	Productivity factor
i_p^f	Foreign aid
n	Population growth rate
r	Borrowing rate
u_1	Fraction of h used to support the production of k
$1-u_1$	Fraction of h used to build new h
v_1	Fraction of g used to support the production of k
v_2	Fraction of g used to build new h in the form of schools
v_3	Fraction of g used to build new h in the form of medical centers
α_1	Share of public resources used for the building of new g
α_2	Share of public resources used for transfers and public consumption
α_3	Share of public resources used for the functioning of the administration
α_4	Share of public resources used for debt services
α	Elasticity of GDP w.r.t. k
β	Elasticity of GDP w.r.t. h
γ	Elasticity of GDP w.r.t. g
δ_k	Depreciation of private capital (k)
δ_g	Depreciation of public capital (g)
δ_h	Depreciation of human capital (h)
ε_1	Elasticity of h w.r.t. existing h
ε_2	Elasticity of h w.r.t. g in the form of schools
ε_3	Elasticity of h w.r.t. g in the form of medical centers
η	Weight of transfers and public consumption in utility
ρ	Discount rate
U	Utility/welfare

The amount of resources absorbed by the public sector, e_p, is used for public infrastructure ($i_p = \alpha_1 e_p$), transfers and public consumption ($c_p = \alpha_2 e_p$), and public administration ($tr = \alpha_3 e_p$). The latter has neither utility nor productive effects, but possesses public goods features and is necessary for the functioning of the state. Finally, for the debt service we have $d = (1-\alpha_1-\alpha_2-\alpha_3)e_p = \alpha_4 e_p$ [6].

When public investment is turned into public capital, we can think of three uses of the public capital: First, there exists public capital which raises productivity of market production, such as transport systems (roads, bridges, harbors) and utilities (for example water supply). In the model, that part is $v_1 g$. The other fraction of public capital is used for facilitating the formation of human capital via health services, $v_2 g$, and education, $v_3 g$. In our view, the formulation with one public capital stock, which is divided (and subdivided) between use in production and human capital formation, is sufficient to study the composition effect of public expenditure.

Note that for the government budget constraint we can write $G = i_p^f + (\alpha_1+\alpha_2+\alpha_3+\alpha_4)e_p$, with $i_p^f + \alpha_1 e_p$ earmarked for public investment and $\alpha_4 e_p$ for debt service. In the steady state then we have $i_p^f + \alpha_1 e_p = (\delta_g + n)g$. The flow of expenditure should be properly subdivided in spending for infrastructure, health, and education, according to the fractions v_1, v_2 and v_3 [7]. Moreover, note that the government revenue is also $T = i_p^f + e_p$, but what can be used for domestic spending is only $T - \alpha_4 e_p$, since $\alpha_4 e_p$ flows out for debt service. The primary surplus of the government budget is $\alpha_4 e_p$.

We presume a planner solves the system of equations below to derive the social optimum:

$$\max_{c, e_p, u_1} \int_0^\infty e^{-(\rho-n)t} \left(\frac{(c(\alpha_2 e_p)^\eta)^{1-\sigma}}{1-\sigma} - 1 \right) dt \qquad (1)$$

subject to

$$\dot{k} = Ak^\alpha (u_1 h)^\beta (v_1 g_1)^\gamma - c - e_p - (\delta_k + n)k \qquad (2)$$

$$\dot{h} = ((1-u_1)h)^{\varepsilon_1} (v_2 g)^{\varepsilon_2} (v_3 g)^{\varepsilon_3} - \delta_h h \qquad (3)$$

$$\dot{g} = i_p^f + \alpha_1 e_p - (\delta_g + n)g \qquad (4)$$

$$\dot{b} = (r-n)b - (1-\alpha_1-\alpha_2-\alpha_3)e_p \qquad (5)$$

Note that for simplicity we assume an additively separable utility function $U(\cdot) = \ln c + \eta \ln(\alpha_2 e_p)$. Note also that equations (2) - (5) describe the constraints to

[6] *Note that we can allow for net borrowing. If we define net borrowing as i_p^b, earmarked for public investment, we will, then, have to add i_p^b in equations (4) and (5) as additional borrowing. We can think of i_p^f in equation (4) as representing both foreign aid and net borrowing. In the steady state debt, b, and the debt-to-income ratio, b/y, will correspondingly fall, since the maximal sustainable debt level will decrease as more debt is taken on.*

[7] *We might think that the annual revenue, earmarked for $i_p^f + \alpha_1 e_p$, is handed over to a trust fund that manages the public current and capital expenditure related to infrastructure, health and education.*

the growth of private capital, human capital, public capital and debt. Since we only consider an exogenous growth model in this version, $\alpha + \beta + \gamma \leq 1$, e.g. $\alpha = \beta = \gamma = 0.33$. One could introduce here some externalities, as in Greiner et al. (2005, ch.3), to obtain endogenous growth.

In a further extension we could allow for foreign borrowing (for public investment) by adding a positive term to equations (4) and (5). Since this borrowing will be constant and permanent, the level of permissible debt level will fall. However, since income will increase, the debt-to-income ratio b/y will also fall. This is exactly what we expect from a numerical exercise[8].

Our formulation of public debt implies that public debt is foreign public debt. New borrowing from abroad can be allowed for[9]. Interest payments on existent foreign debt (for example foreign bonds) is rb (at world interest rate). Repayments and interest payments do not go to the domestic household sector, but go to foreigners. The usual transversality condition is assumed to hold (see section 3.2).

Next we explore the stationary state of the model in its form of equations (1) - (5). We employ the Hamiltonian to sketch a solution of the model. The Hamiltonian is

$$\begin{aligned} H(c,e,k,h,g) &= ln\ c + \eta ln(\alpha_2 e_p) \\ &+ \lambda_1 \left[Ak^{\alpha}(u_1 h)^{\beta}(v_1 g)^{\gamma} - c - e_p - (\delta_k + n)k\right] \\ &+ \lambda_2 \left[((1-u_1)h)^{\beta_1}(v_2 g)^{\beta_2}(v_3 g)^{\beta_3} - (\delta_k h\right] \\ &+ \lambda_3 \left[i_p^f + \alpha_1 e_p - (\delta_g + n)g\right] \\ &+ \lambda_4 \left[(r-n)b - (1-\alpha_1-\alpha_2-\alpha_3)e_p\right] \end{aligned} \quad (6)$$

with λ_1, λ_2, λ_3, λ_4 the co-state variables. The first order conditions for the two choice variables, c and e_p, are:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow c^{-1} = \lambda_1 \quad (7)$$

$$\frac{\partial H}{\partial e_p} = 0 \Rightarrow \eta e_p^{-1} = \lambda_1 - \lambda_3 \alpha_1 - \lambda_4(1-\alpha_1-\alpha_2-\alpha_3) \quad (8)$$

[8] *Note that this term does not have to be constant. It suffices if the term is positive on average for the results to hold.*

[9] *See footnote 3.*

For the co-state variables we have

$$\dot{\lambda}_1 = \lambda_1(\rho - n) - \frac{\partial H}{\partial k} = \lambda_1(\rho - n) - \lambda_1 \left[A\alpha k^{\alpha_1}(u,h)^{\beta}(v_1 g)^{\gamma} - (\delta_k + n)\right] \tag{9}$$

$$\begin{aligned}\dot{\lambda}_2 = \lambda_2(\rho - n) - \frac{\partial H}{\partial h} = \lambda_2(\rho - n) \quad &- \quad \lambda_1\left[Ak^{\alpha}\beta(u,h)^{\beta-1}u_1(v_1 g)^{\gamma}\right] \\ &- \quad \lambda_2\left[\varepsilon_1((1-u_1)h)^{\varepsilon_1 - 1}(1-u_1)((1-v_1)g)^{\varepsilon_2} - \delta_h\right]\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{\lambda}_3 = \lambda_3(\rho - n) - \frac{\partial H}{\partial h} = \lambda_3(\rho - n) \quad &- \quad \lambda_1\left[Ak^{\alpha}(u_1 h)^{\beta}\gamma(v_1 g)^{\gamma-1}u_1 \cdot v_1\right] \\ &- \quad \lambda_2\left[((1-u_1)h)^{\varepsilon_1}((1-v_1)g)^{\varepsilon_2 - 1}\varepsilon_2(1-v_1)\right] + \lambda_3(\delta_g + n)\end{aligned} \tag{11}$$

$$\dot{\lambda}_4 = \lambda_4(\rho - n) - \frac{\partial H}{\partial b} = \lambda_4(\rho - n) \quad - \quad \lambda_4(r - n) \tag{12}$$

Equations (7)-(12), which are derived from the two first order conditions with respect to the choice variables c and e_p and the four equations for the co-state variables λ_1, λ_2, λ_3, and λ_4, give us, together with the four state variable equations (2)-(5), a system of ten equations in ten variables $\{c, e, k, h, g, b, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$. Writing eqns. (7) and (8) as

$$0 = \lambda_1 - c^{-1} \tag{13}$$

$$0 = \lambda_1 - \lambda\alpha_1 - \eta e_p^{-1} - \lambda_4(1 - \alpha_1 - \alpha_2 - \alpha_3) \tag{14}$$

and setting the differential equations (2)-(5) as well as (9)-(12) equal to zero, we can obtain a stationary state $\{c^*, e^*, k^*, h^*, g^*, b^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*\}$. Plugging these into the production function $y = Ak^{\alpha}(u_1 h)^{\beta}(v_1 g)^{\gamma}$, we can also obtain the per capita income, y^*, at the stationary state.

Given our specification of human capital and its use, we can study the effect of the allocation of human capital on per capita income and the other macroeconomic variables. Furthermore, varying our parameter set v_1, v_2 and v_3, we can explore the impact of different public expenditure compositions on the steady-state outcomes of per capita income and the other variables. A further effect on per capita income is predicted to be caused by a change of the parameter A and of foreign aid, $i_p^f > 0$. Overall, from all of those comparative static studies we expect to obtain information on per capita income, consumption and the other macro variables.

Specifying all parameters, the following numerical stationary solution is obtained from necessary optimality conditions using the computer software Mathematica[10 11 12]:

[10] *The parameter values used are:* $n = 0.015$, $\rho = 0.03$, $\delta_k = 0.075$, $\delta_h = 0.075$, $\delta_g = 0.05$, $\alpha = \beta = \gamma = 0.33$, $u_1 = 0.85$, $v_1 = 0.8$, $v_2 = 0.1$, $v_3 = 0.1$, $\varepsilon_1 = \varepsilon_2 = 0.2$, $\varepsilon_3 = 0.25$, $\alpha_1 = 0.1$, $\alpha_2 = 0.7$, $\alpha_3 = 0.1$, $i_p^f = 0.05$, $A = 1$, $\eta = 0.1$, $r = 0.07$.

[11] *The* Mathematica *code is available at www.newschool.edu/gf/cem.*

[12] *Note that these values are dimensionless and are calibrated in such a way as to fit the data. Economic implications are derived in later sections, where the effects of comparative changes are explored.*

$$k^*= 86.59,\ h^*= 21.97\ ,\ g^*=17.86\ ,\ b^*= 20.20,\ y^*= 27.55,\ \frac{b^*}{y} = 0.73$$

With this solution technique we can undertake the above described comparative static analyses and some important calibrations, see sects. 4 and 5[13].

3.2 Fiscal Policy to Ensure Debt Sustainability[14]

The model of section 3.1 permits government to borrow from capital markets, in particular to borrow from abroad in order to undertake public investment. In this section, we want to explore the effects and implications of government borrowing from capital markets and briefly discuss the sustainability of fiscal policy when borrowing is allowed for[15 16].

Let us assume that the government has borrowed from abroad for undertaking public investment. The implication of this type of expenditure is that along the transition path, this will raise the growth rate of public infrastructure and lead to a higher level of public capital. This higher level brings a distortion into the model by raising the marginal product of private capital. As a consequence, the investment share is increased and the growth rate of consumption rises implying higher welfare after a sufficiently long adjustment period. Note that this also leads to higher growth of physical and human capital. However, in our growth model these growth effects only hold on the transition path. In the long-run, higher public investment leads to higher levels of output and consumption but does not affect the growth rate of endogenous variables.

Since here borrowing from abroad equals loans or issuing of bonds to foreigners, the government must pay it back plus interest payments. Debt repayment is represented by $\alpha_4 e_p$. More concretely, the government has to stick to the inter-temporal budget constraint which can be written as

$$b(0)=\int_0^\infty e^{-\int_0^\tau (r(\mu)-n)d\mu} s(\tau)d\tau \Leftrightarrow \lim_{t\to\infty} e^{-\int_0^t (r(\tau)-n)d\tau} b(t)=0 \qquad (15)$$

where s denotes the primary surplus which, in our model, is given by $\alpha_4 e_p$. The first term in (15) states that per-capita government debt at time zero must equal the discounted stream of future primary surpluses. This implies that the government must

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13 *Allowing for additional borrowing by adding a constant in eqns. (4) and (5) and setting this borrowing at the same size as foreign aid, $i_p^b = 0.05$, we obtain the following numerical values: $k^*= 88.67$, $h^*= 22.36$, $g^* = 18.43$, $y^*= 28.21$, $b/y^*= 0.67$. As one can observe, b and b/y fall, indicating a fall in permissible debt.*

14 *Since this section treats empirical matters, its notation is changed in order to comply with readily available data (e.g. G now refers to primary expenditure and not public capital as in the rest of the paper).*

15 *Again, note that in case of net borrowing a constant needs to be added to equation (5) and the foreign aid parameter i_p^f in equation (4) is supposed to be corrected for the availability of new investment funds for public capital.*

16 *In Greiner et al. (2005), ch. 6, more particular model versions are developed with government borrowing for specific government expenditures.*

run primary surpluses, if it starts with a positive stock of debt. Equivalent to this formulation is the requirement that discounted debt converges to zero asymptotically. It should be mentioned that in theoretical exogenous growth models the inter-temporal budget constraint is met, provided the interest rate exceeds the population growth rate on average, because per-capita debt converges to a finite value. Nevertheless, it is important for real world economies to check for rules whether fiscal policies of countries are such that they fulfill the inter-temporal budget constraint or not.

One possible rule for sustainability is to require that the government adjusts the primary surplus to GDP ratio to variations in the debt-GDP ratio. To see that a positive linear dependence of the primary surplus ratio[17] on the debt ratio can guarantee sustainability, we assume that the primary surplus ratio is given by

$$\frac{s(t)}{y(t)} = \alpha + \beta(t)\frac{b(t)}{y(t)} \tag{16}$$

with $y(t)$ per-capita GDP. $\beta(t)$ determines how strongly the primary surplus to GDP ratio reacts to changes in the public debt-GDP ratio and α, which is assumed to be constant, can be interpreted as a systematic component determining how the level of the primary surplus reacts to a rise in GDP. α can also be seen as representing other constant variables which affect the primary surplus to GDP ratio.

Using equation (16), the differential equation describing the evolution of public debt can be rewritten as

$$\dot{b}(t) = (r(t)-n)b(t) - s(t) = (r(t)-n-\beta(t))b(t) - \alpha y(t). \tag{17}$$

Solving this differential equation and multiplying both sides by $e^{-\int_0^t (r(\tau)-n)d\tau}$ to get the present value of public debt yields

$$e^{-\int_0^t (r(\tau)-n)d\tau} b(t) = e^{-\int_0^t \beta(\tau)d\tau}\left(b(0) - \alpha y(0)\int_0^t e^{\int_0^\tau (\beta(\mu)-r(\mu)+n+\gamma(\mu))d\mu} d\tau\right), \tag{18}$$

with γ the growth rate of GDP. Equation (18) shows that a positive value for β on average, so that $\int_0^t \beta(\tau)d\tau$ converges to plus infinity asymptotically, is necessary for sustainability. It is also sufficient, if $r(\mu)-n-\gamma(\mu)$ is positive on average[18].

It should be noted that a positive value for $r-n-\gamma$ characterizes a dynamically efficient deterministic economy. If the economy is stochastic, however, this need not necessarily

[17] Note in the subsequent part we solely focus on the sustainability issue and ignore the feedback effects of a changing surplus on the other spending categories.

[18] This holds because l'Hôpital gives the limit of the second term in (18) multiplied by $e^{-\int_0^t \beta(\tau)d\tau}$ *as* $e^{-\int_0^t (r(\mu)-n-\gamma(\mu))d\mu}/\beta$

hold for the economy to be dynamically efficient. Nevertheless, testing the reaction of the primary surplus to variations in the debt ratio is reasonable because a positive reaction is necessary for sustainability as mentioned above. Further, this test yields insight as to how governments deal with public debt and, thus, shows how important the goal of stabilizing debt is for the government.

One advantage of the sustainability test just presented is to be seen in the fact that it does not depend on the interest rate, in contrast to other tests where the assumed interest rate may be crucial as to the outcome whether a given policy is sustainable or not. In addition, the proposed rule is intuitively plausible from an economic point of view. If a government runs a deficit, it has to run a primary surplus in the future to pay back the deficit. Otherwise, sustainability is not given. In our model economy, this implies a withdrawal of resources from the economy. Therefore, a deficit financed increase in public investment is beneficial for the economy, if the gain in productivity is sufficiently high to cover the interest payments and the loan. A detailed discussion of different types of sustainability tests and how the above suggested sustainability rule can be implemented is undertaken in section 6 of the paper. Next we explore the change of the basic structure of the model and its impact on the macroeconomic variables such as output, consumption, capital stock and debt.

4. Exploring the Basic Structure of the Model

We want to explore the effect of foreign aid, i_p^f, the productivity factor, A, and the fraction of human capital used in market goods production, u_1, on the steady-state variables of three group of countries classified by income level. We follow the World Bank's 2006 paper to the Development Committee in classifying countries in low-income, lower-middle-income, and upper-middle-income groups. Due to the quality of the data, however, only a reduced list of countries is used for the calibration exercise[19]. We employ only data for the time period 1994 to 2004. First, we will calibrate the effect of foreign aid, i_p^f [20]. By looking at actual data of foreign aid per capita, we are able to determine a range in which our parameters can vary. Before we can evaluate the effect of foreign aid, productivity factor, and the fraction of human capital used in market goods production on per capita income, we need to calibrate our above-mentioned model (1) to (5) such that we can roughly reproduce the differences in per capita income across the three groups of countries. Since, as discussed above, there are many forces of growth causing the differences in per capita income, we adjust the productivity level of the three groups, the parameter A, such that we roughly

[19] *See Appendix A.4 for the complete list of countries.*

[20] *Note that in the following we presume that there is no new borrowing, so that i_p^f is solely foreign aid.*

obtain the actual differences in per capita income, normalizing the parameter, A, of the lowest per capita income group. Therefore A is set to 1 for the lowest income group. We hereby take the composition of total public expenditure, α_1, α_2, α_3, and α_4, as given for each group as we find them in the data. Now we employ our *Mathematica* program to obtain stationary state solutions for the three groups of countries[21]. This is equivalent to a comparative static analysis for the three country groups. Our exercise consists in keeping the parameter, A, constant for each group of countries and equal to its calibrated value. Then we let foreign aid, i_p^f, vary during that period and see how such a change affects the steady-state values k, h, g, c, e_p, y, b/y and U. Such an exercise was conducted for the low-, lower-middle-, and upper-middle income groups. In the next step we vary the productivity parameter, A, for each group of countries, employing for each country group the A that we used to calibrate the model to obtain the actual differences in per capita income, and increasing it gradually. The parameter, A, is set equal to 1, 1.18, and 1.45 for the low-, lower-middle-, and upper-middle income groups respectively. Finally, we vary u_1, the fraction of human capital used in market goods production, from 0.1 to 0.8, keeping foreign aid equal to its 10-year average, and the other parameters constant.

4.1 Effects of Foreign Aid per Capita, i_p^f

The comparative static results for all income groups show that k, h, g, c, e_p, y, b/y, and U are linear in i_p^f and there is clearly a positive relationship between the level of foreign aid and all the variables, except for public resources per capita, e_p, and debt per GDP, b/y, which show a negative relationship with respect to foreign aid, i_p^f [22]. As the foreign aid goes up the public resources per capita falls slightly. The results suggest that any increase in foreign aid per capita would increase k, h, g, c and y but would reduce only very slightly the optimally chosen public resources, e_p. Such a negative relationship could be explained by the fact that public resource is a result of a lump-sum taxation and as more aid flows in, there is less and less incentive to tax, and as a result, a decrease in public resources, e_p. A similar argument can be made for debt per GDP, b/y. As more aid flows in, there is less and less incentive to borrow, and as a result, there would be a fall in b/y as i_p^f increases. Actual data for the period 1994-2004 shows that the per capita foreign aid went from 2 to 4. To conduct our comparative static exercises, we increased i_p^f by increments of 0.2 from 2 to 4, which is the range provided by the actual data[23]. The results of our comparative static exercises for the low-income group are summarized in Table 4 in appendix A.5. The results for the remaining two

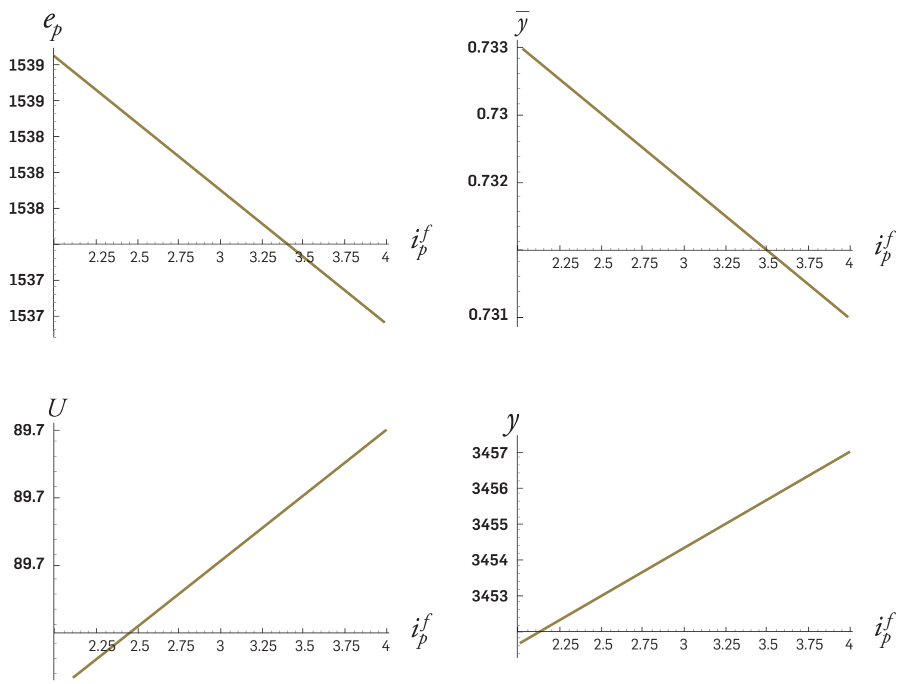
[21] *The* Mathematica *code is available at www.newschool.edu/gf/cem.*

[22] *This paper does not address the problems of moral hazard and inefficiency that often characterize aid inflows into poor governance environments. Waste, corruption, and inefficiency would clearly weaken the effects of foreign aid on growth and welfare.*

[23] *Recent literature has studied the effect of foreign aid on per capita income in country by country studies (see Easterly (2006)) or in cross-section studies (see Reddy and Minoiu (2006)). The latter study finds significant and large effects.*

income groups can also be found in the appendix A.5. Figure 1 below shows the relationship between i_p^f and y, U, b/y, and e_p respectively for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups[24].

Figure 1. **Effects of Foreign Aid, i_p^f, on the Steady State Variables (Low-Income Group)**



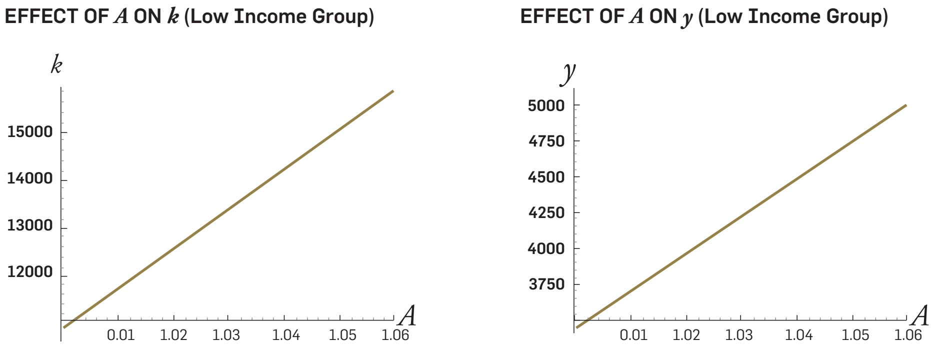
4.2. Effect of A, the productivity factor

The comparative static results for all income groups show that k, h, g, c, e_p, y, b/y and U are linear in A and there is clearly a positive relationship between the level of foreign aid and all the variables. The results suggest that any increase in A would increase k, h, g, c, e_p, y, b/y and U. Because the low-income group is our reference group, A is increased from 1 to 1.06, and the effect on the steady-state variables is observed. As one might predict, results show that there is a positive linear relationship between A and all the steady-state values of the variables, with the fastest increase in k, y and g, relative to the other variables. The results of our comparative static exercises for the low-income group are summarized in Table 5 in appendix A.5. The results for the other two income groups can also be found in the appendix. Figure 2 below shows the rela-

[24] *Note that the effect of an increase of A has rather large effects on output. But also note that an increase of our A by 6% would correspond to an increase in the efficiency of labor by roughly 20%, since we have written A as outside the production function, namely as exogenous technical change affecting total factor productivity.*

tionship between A and y, and k respectively for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups.

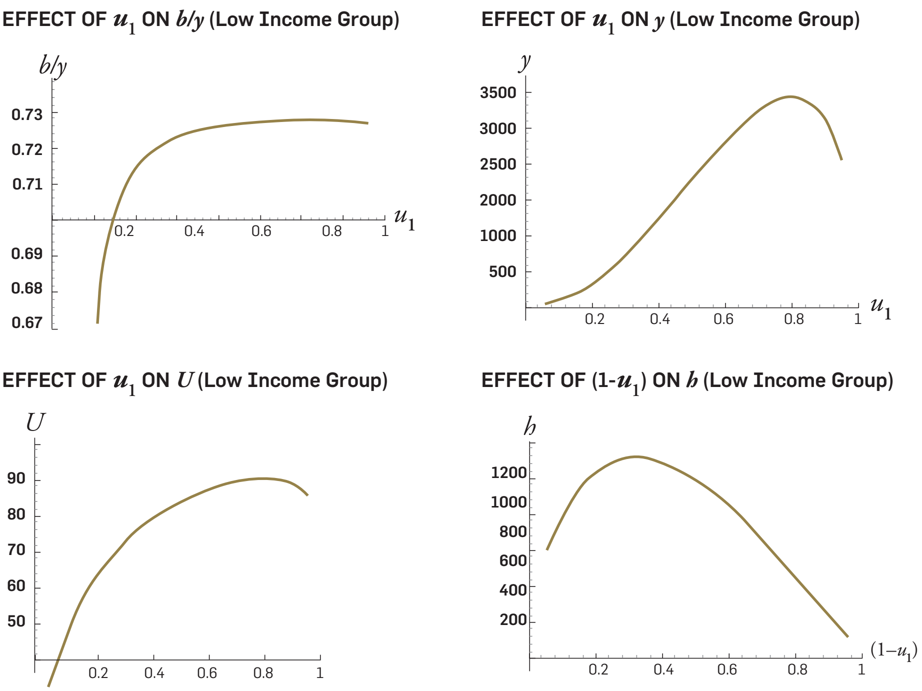
Figure 2. Effects of the Productivity Factor A on the Steady State Variables (Low-Income Group)



4.3 Effects of the Fraction of Human Capital Used in Market Goods Production, u_1

To analyze the effect of the parameter u_1, it was increased from 0.05 to 0.95 with increments of 0.05, and the steady-state values corresponding to the different values of the parameter were recorded. The comparative static results on u_1 for all income groups show a hump-shape, non-linear relationship, with respect to k, h, g, c, e_p, y, b/y and U. With respect to the hump-shape relationship, we can make the following observations: First, we observe a positive relationship between u_1 and k, h, g, c, e_p, y, b/y and U for $u_1 \leq 0.8$, and a negative relationship for $u_1 \geq 0.8$. We observe increasing returns for k, h, g, c, e_p, y, b/y, when $u_1 \leq 0.3$ and decreasing returns when $u_1 \geq 0.3$. As for U, and b/y, we always observe decreasing returns for all values of u_1. Second, as to human capital, h, we observe a positive relationship between $1-u_1$ and h when $1-u_1 \leq 0.3$, and a negative relationship for $1-u_1 \geq 0.3$. Finally, it was observed in our exercises that the nature, and form of the hump-shape relationship depend on the choice of our parameters. For instance, we chose ε_1, ε_2, and ε_3 to sum up to less than 1, indicating decreasing returns. When the above parameters are chosen such that they sum up to more than 1, both the nature of the relationship and the optimal point for u_1 change. The results of our comparative static exercise for the low-income group are summarized in Table 6 in appendix. The results for the other two income groups can also be found in the appendix A.5. Figure 3 above shows the relationship between u_1 and y, U, h, and b/y, and between $1-u_1$ and h, for the low-income group. Similar results are observed for the lower-middle and upper-middle income groups.

Figure 3. **Effect of the Fraction of Human Capital Used in Market Goods Production, u_1, on the Steady State Variables (Low-Income Group)**



The observation that u_1 , has a non-linear, hump-shaped relationship with all the variables suggests a decreasing return effect so that, beyond a point, any increase in the fraction of resources allocated to one area takes away resources from other areas that may indeed contribute more to growth. Furthermore, the hump-shaped form of the effect of $1{-}u_1$ on per capita human capital, h, implies that any increase in the parameters would first increase h, but beyond the 30% threshold, the effect becomes negative. Such a reversal of the relationship could signal the fact that as too much human capital is devoted to human capital production, there is less and less of it available for market goods production.

5. Exploring the Effect of Public Expenditure Composition

In this section the model is calibrated for different expenditure structures and the effects of changes in the composition of public investment expenditure are explored for the three country groups. As above, the classification in low-, lower-middle- and upper-middle-income countries is retained[25]. Here, again, only the results of the low-

[25] *See appendix A.4 for the complete list of countries.*

income groups are reported. For the results of the other two country groups, see Appendix A.6. The data on public expenditure is obtained from the IMF's Government Finance Statistics. Since these are too detailed for our analysis, the different expenditure categories are summarized to match the following model parameters: public investment in infrastructure, $\alpha_1 v_1$, education, $\alpha_1 v_2$, and health, $\alpha_1 v_3$, public transfers, α_2, public consumption, α_3 and debt services, $\alpha_4 = (1-\alpha_1-\alpha_2-\alpha_3)$[26]. Note that the values reflect the averages for all countries of one category.

By plugging the obtained numerical values ($\alpha_1 = 0.4035$, $\alpha_2 = 0.0623$, $\alpha_3 = 0.4342$) into the model, setting $\rho = 0.03$, $\delta_k = 0.075$, $\delta_h = 0.075$, $\delta_g = 0.05$, $\alpha = \beta = \gamma = 0.33$, $u_1 = 0.85$, $\varepsilon_1 = \varepsilon_2 = 0.2$, $\varepsilon_3 = 0.25$, $r = 0.08$, $\eta = 0.1$ and using the values for A and i_p^f that were found in the previous section ($A = 1$, $n = 0.0195$ $i_p^f = 3.361$), one can simulate the effect of changes in the composition of public expenditure on the stationary states by letting v_1 , v_2 and v_3 vary[27].

In the following this is done for the low-income country group[28]. Although all graphs are presented in the 3-dimensional space such that the interaction of all three investment expenditure parameters can be observed at the same time, the stationary states for shifts from public investment in health, $\alpha_1 v_2$, to public investment in education, $\alpha_1 v_3$, are of particular interest[29].

Generally, the results suggest that, in order to maximize the growth of per capita income, more than half of public investment expenditure should be on infrastructure while the residual should be roughly equally allocated to investments that support production of human capital. Figures 4 to 12, all of which are hump-shaped, depict the effects of changes in the investment expenditure composition on the various model parameters. Figures 4 to 9 show all possible investment expenditure compositions for income, y , capital, k, public capital, g, public debt, b, consumption, c and total public resource absorption, e_p , are qualitatively the same. Only the graphs for human capital, h, utility, U, and debt/income ratio, b/y, differ in their behavior. In Figure 12 one can see that human capital can be increased beyond its welfare- and growth-maximizing level by setting v_2=0.30 and v_3=0.20 (and v_1=0.5). At this point the maximal h is reached, however, the benefits of its increase do not offset the costs of higher depreciation. Figure 10 depicts the changes in utility over the range of v_1, v_2 and v_3. Whilst the curvature of U is less pronounced than in the previous graphs, it is

[26] *See appendix A.3 for details.*

[27] *Note that described the numerical values for ρ, δ_k, δ_h, δ_g, α, β, γ, u, ε_1, ε_2, ε_3, α_1, α_2, α_3 and η remain the same for the remainder of this section.*

[28] *Similar results hold for the other two groups. Complete sets of graphs for the other groups are listed in Appendix A.6.*

[29] *Note that $v_1 + v_2 + v_3 = 1$. always has to hold which restricts the plane to a triangular space. By fixing v_2 or v_3 the effects of changes in v_1 are also always observable.*

hump-shaped and possesses an interior maximum which lies at general maximum, i.e. v_1=0.65 , v_2=0.15 and v_3=0.20. The shape of the debt to income ratio, b/y, differs completely from the other variables as it does not possess an interior maximum. As one can see in graph 11, the curvature of b/y is strictly concave and attains its maximum value at the origin, i.e. v_2 =v_3=0 (and v_1=1). The fact that is, however, more interesting is that the ratio is approximately stable through most of the range of v_2 and v_3.

Sustainability problems due to shifts in the investment expenditure composition do not occur in the long run. The aim of this section was to conduct a comparative static exercise for changes in the composition of public investment expenditure. The results show that in the long run the welfare and growth maximum lies at a composition of roughly v_1=0.65, v_2=0.15 and v_3=0.20. Debt sustainability problems did not occur throughout the possible range of v_1, v_2 and v_3. Yet, overall to the issue of debt sustainability we have presumed here a very passive debt policy as indicated in equation (5), which assumes that new debt is not issued and the repayment follows a fixed rule, namely $e_p \alpha_4$. If we allow for new debt issue, as discussed in section 3.1, the debt to output ratio, b/y is likely to increase.

Figure 4.
Effects of Changes on Income *y*

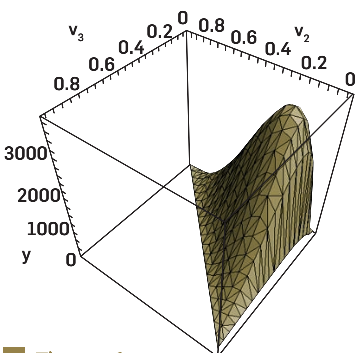


Figure 5.
Effects of Changes on Capital *k*

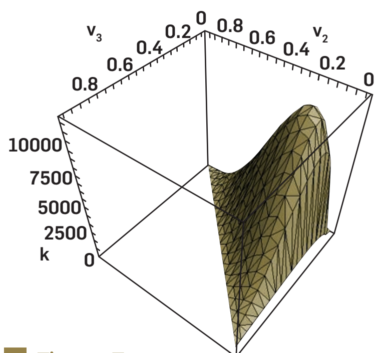


Figure 6.
Effects of Changes on Public Debt *b*

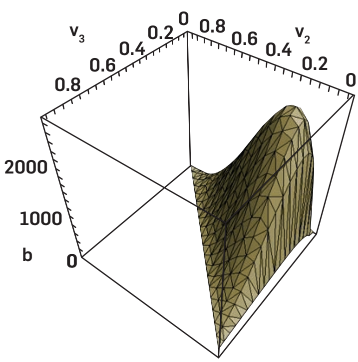


Figure 7.
Effects of Changes on Public Capital *g*

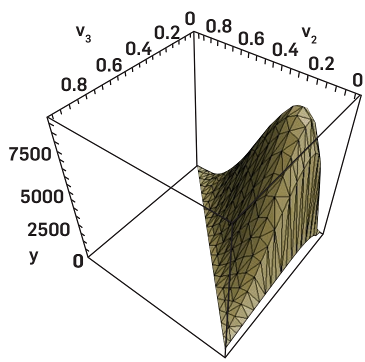


Figure 8.
Effects of Changes on Consumption c

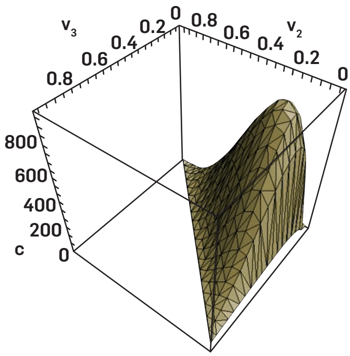


Figure 9.
Effects of Changes on Public Absorption e_p

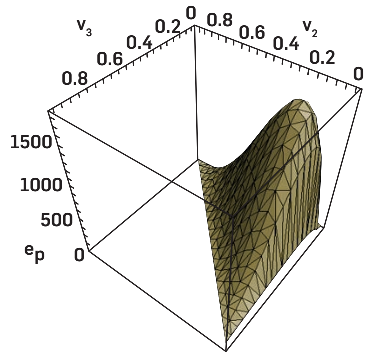


Figure 10. **Effects of Changes on Debt to Income Ratio b/y**

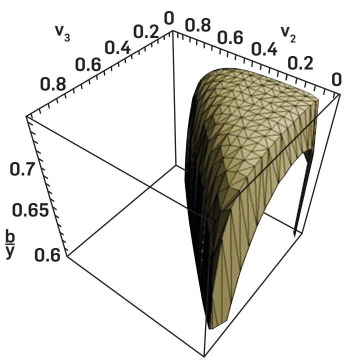


Figure 11.
Effects of Changes on Utility U

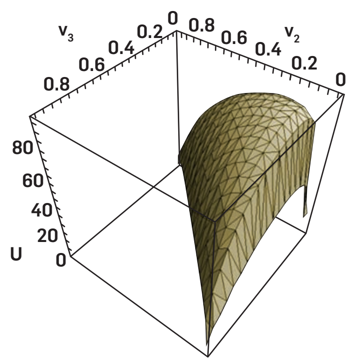
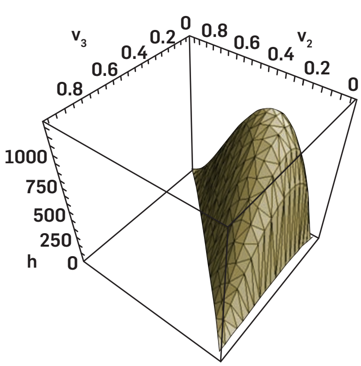


Figure 12. **Effects of Changes on Human Capital h**



6. Testing Debt Sustainability[36]

In section 3.2 we modeled the inter-temporal budget constraint in the context of our model but we did not pursue this question further in our calibration studies in section 4 and 5. If one studies particular countries (or country groups) from a time series perspective, it is an important empirical question whether certain public policies for growth stay within the bounds of a sustainable fiscal policy. Here we briefly discuss tests for sustainability of fiscal policies and suggest a version that has been tested for advanced countries. We will then suggest sustainability tests that can be undertaken for specific country studies[31].

If one studies the effect of fiscal policy on economic growth, one needs to check, as indicated in section 3.2, whether the government fulfills the inter-temporal budget constraint. As shown in section 3.2, in economic terms, this constraint states that public (net) debt at time zero must equal the expected present value of future present-value primary surpluses. In general terms, this requirement is also often referred to as the no-Ponzi game condition. In a general form we can write this as follows. Neglecting stochastic effects and assuming that the interest rate is constant, the intertemporal budget constraint can be written as

$$B(0) = \int_0^\infty e^{-r\tau} S^p(\tau) d\tau \,, \tag{19}$$

with r the constant interest rate, $B(0)$ public debt at time zero and S^p the primary surplus[32].

Equivalent to equation (19) is the following equation

$$\lim_{t \to \infty} e^{-rt} B(t) = 0, \tag{20}$$

with $B(t)$ public debt at time t, stating that the present value of public debt converges to Zero for $t \to \infty$.

In the economic literature numerous studies exist which explore whether (19) and (20) hold in real economies[33]. For example, Hamilton and Flavin (1984) suggest testing for the presence of a bubble term in the time series of public net debt which would indicate that a given fiscal policy is not sustainable. Trehan and Walsh (1991) pro-

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[30] *As in section 3.2 the notation of this section is changed in order to comply with empirically available data.*

[31] *For a detailed discussion of sustainability tests see Greiner et al. (2005).*

[32] *For further derivations see e.g. Blanchard and Fischer (1989), ch. 2.*

[33] *See e.g. Hamilton and Flavin (1984), Kremers (1988), Wilcox (1989), and Trehan and Walsh (1991).*

posed to test whether the budget deficit is stationary or to test whether the primary budget deficit and the public debt series are co-integrated and $(1-\lambda L)S_t^p$ is stationary, with $0 \leq \lambda < 1 + r_t$. Another test, proposed by Wilcox (1989), is to test whether the series of undiscounted debt displays an unconditional mean of zero. If this holds the inter-temporal government constraint will be fulfilled, because the inter-temporal budget constraint requires the discounted debt to converge to zero[34]. This implies that all government debt will be repaid at some point in time.

Moreover, another aspect of these tests which has given rise to criticism is that those tests need strong assumptions, because the transversality condition involves an expectation about states in the future that are difficult to obtain from a single set of time series data and because assumptions about the discount rate have to be made[35].

Alternative tests check solely if in the long run some debt-to-income ratio remains stationary. A test procedure which circumvents the problems associated with the above first type of test focuses on the time series of the debt ratio, i.e. on the ratio of public debt-to-GDP. If this series is constant the inter-temporal budget constraint is fulfilled for dynamically efficient economies. To see this let $B/Y = c_1$ be the constant debt ratio, with Y denoting GDP and c_1 a positive constant. Inserting $B/Y = c_1$ in (20) yields

$$\lim_{t\to\infty} c_1 Y_0 e^{(\gamma - r)t} = 0, \qquad (21)$$

for $\gamma < r$, with $\gamma > 0$ the constant growth rate of GDP. The condition $\gamma < r$ characterizes a dynamically efficient economy and is likely to hold in real economies. For example, in EU countries this seems to be obvious if one compares interest rates with GDP growth rates. But even the US, where growth rates have exceeded interest rates on safe government bonds since the 1990s, is a dynamically efficient economy[36]. Therefore, for advanced countries we may limit our considerations to the case of dynamic efficient economies and assume that the discount rate of government debt exceeds the GDP growth rate[37]. For developing countries, like the low- and lower-middle income countries of this study, this would have to be explored.

However, testing for stationarity of the debt-to-GDP ratio is characterized by some shortcomings, too. It is difficult to distinguish between a time series which is stationary about a positive intercept and one that shows a trend. This holds because standard unit root regressions have low power against autoregressive alternatives if the AR co-

[34] *As to these tests applied to Germany see Greiner and Semmler (1999).*

[35] *See e.g. Bohn (1995,1998).*

[36] *For details of such conditions see Abel et al. (1989).*

[37] *In dynamically inefficient economies the government budget constraint is irrelevant because in that case the government can play a Ponzi game.*

efficient is close to one. As a consequence, the hypothesis that a given fiscal policy is sustainable has been rejected too easily.

Therefore, Bohn (1998) suggests to test whether the primary deficit-to-GDP ratio is a positive linear function of the debt-to-GDP ratio. If this holds, a given fiscal policy will be sustainable. As discussed in section 3.2 the intuitive reasoning behind this argument is that if a government raises the primary surplus as the public debt ratio rises, it takes a corrective action which stabilizes the debt ratio and makes public debt sustainable. Before one can undertake empirical tests which apply this concept we need to advance some theoretical considerations about the relevance of this test. We limit our considerations to deterministic economies.

Assuming, as in section 3.2, that the primary surplus to GDP ratio depends on a constant and linearly on the debt-to-GDP ratio, this variable can be written as

$$\frac{S^p(t)}{Y(t)} = \frac{T(t)-G(t)}{Y(t)} = \alpha+\beta\left(\frac{B(t)}{Y(t)}\right), \qquad (22)$$

with $T(t)$ tax revenue at t, $G(t)$ public spending at t, α and β are constants which can be negative or positive. α is a systematic component which determines how the level of the primary deficit reacts to variations in GDP. α can also be interpreted as other (constant) economic variables which affect the surplus ratio. The coefficient β can be called a reaction coefficient since it gives the response of the primary surplus ratio to an increase in the debt ratio. Inserting (22) in the differential equation, giving the evolution of public debt, the latter equation is then given by

$$\dot{B}(t)=rB(t)+G(t)-T(t)=(r-\beta)B(t)-\alpha Y(t). \qquad (23)$$

Solving this equation we get

$$B(t)=\left(\frac{\alpha}{r-\gamma-\beta}\right)Y(0)e^{\gamma t}+e^{(r-\beta)}C_1. \qquad (24)$$

with $B(0)>0$ debt at time $t=0$ which is assumed to be strictly positive and with C_1 a constant given by $C_1= B(0)-Y(0)\alpha/(r-\gamma-\beta)$. Multiplying both sides of (24) by e^{-rt} leads to

$$e^{-rt}B(t)=\left(\frac{\alpha}{r-\gamma-\beta}\right)Y(0)e^{(\gamma-r)t}+e^{-\beta t}C_1. \qquad (25)$$

The first term on the right hand side in (25) converges to zero in dynamically efficient economies and the second term converges for $\beta>0$ and diverges for $\beta<0$. These considerations show that $\beta>0$ guarantees that the inter-temporal budget constraint of the government holds.

Further, with equation (22) the debt-to-GDP ratio evolves according to

$$\dot{b}=b\left(\frac{\dot{B}}{B}-\frac{\dot{Y}}{Y}\right)=b(r-\beta-\gamma)-\alpha. \quad (26)$$

Solving this differential equation, we obtain the debt-to-GDP ratio b as

$$b(t)=\left(\frac{\alpha}{r-\gamma-\beta}\right)+e^{(r-\gamma-\beta)t}\,C_2, \quad (27)$$

where C_2 is a constant given by $C_2=b(0)-\alpha/(r-\beta-\gamma)$, with $b(0)\equiv B(0)/Y(0)$ the debt-GDP ratio at time $t = 0$.

Equation (27) shows that the debt-to-GDP ratio remains bounded if $r-\gamma-\beta<0$ holds. This shows that a positive β does not assure boundedness of the debt-GDP ratio although the intertemporal budget constraint of the government is fulfilled in this case. Only if β is larger than the difference between the interest rate and the GDP growth rate the debt ratio remains bounded. These considerations demonstrate that sustainability of debt may be given even if the debt-to-GDP ratio rises over time - a situation which seems to hold for some countries.

But it must also be pointed out that for too high a level of public debt the government will probably not be able to raise the primary surplus any further. Then, our rule formulated in equation (27) will break down. This holds because, the present value of future surpluses must equal public debt at any finite point in time. So, if the government is not able to raise the primary surplus as public debt rises any longer from a certain point of time, say t_1, onwards, β is zero or even negative. Then, a fiscal policy is sustainable only if the government has succeeded to reduce public debt to zero up to that point of time t_1. This implies that in the long-run the debt-to-GDP ratio must be constant, although it may well rise transitorily. However, it is ultimately an empirical question what the country specific β will be[38].

In this section therefore we suggest some method related to the Bohn method (Bohn, 1998) which allows one to estimate sustainability, but at the same time works, as section 3.2 has indicated, with a time varying reaction of governments to the debt to GDP ratio. This allows one to get some insight using empirical tests about the sustainability of policies. As we have pointed out in section 3.2 our strategy for testing the sustainability of debt has the advantage that the test does not depend on the interest rate which is used to discount public debt as needed in the first type of the above discussed tests.

[38] *This could be estimated separately for each country in country studies.*

Following our setup in section 3.2, the equation suggested to be tested is as follows:

$$s_t^y = \beta_t b_t^y + \alpha^T Z_t + \varepsilon_t, \tag{28}$$

where s_t^y and b_t^y are the primary surplus to GDP and the debt-to-GDP ratio, respectively. Z_t is a vector which consists of the number 1 and of other factors related to the primary surplus and ε_t is an error term which is i.i.d. $N(0,\sigma^2)$.

The idea behind estimating (28) is the tax smoothing hypothesis according to which public deficits should be used to finance transitory government spending, for example higher public spending during recessions. Further, the variables contained in Z_t may differ depending on which country is analyzed. In the US, for example, military spending is a variable which exerts a strong influence on the primary surplus. In European countries social spending plays an important role affecting the primary surplus. For other countries, for example, low-income countries, other spending categories may be relevant.

We want to note, as concerns the primary surplus, one has to distinguish between the primary surplus including the social surplus and the primary surplus exclusive of the social surplus, where social surplus means the surplus of social insurance system. This holds because in some countries deficits of the social insurance system raise the stock of public debt, since the government sector and the social insurance system are not separated, whereas this does not hold for other countries. So, in some European countries the social insurance systems are autonomous and do not borrow in capital markets. For example, if the social insurance system runs deficits, these deficits are either transitory because they must be paid back in the next period or the deficits are covered by reserves from earlier years.

If, on the other hand, the social insurance system has a surplus this does not raise the surplus of the government, but is used as reserve. However, it happens that the government subsidizes the social insurance which leads to a decline in the surplus of the government. This amount, however, is included in the regular surplus or deficit of the government so that taking it into account in the surplus or deficit of the government and adding that of the social insurance would lead to double counting.

These considerations demonstrate that institutional regulations determine which concept for the primary surplus should be used. As we noted above, social security and transfer arrangements in countries of different income groups seem to be different. Our suggested test should therefore be adjusted to each type of income group - or country if country specific fiscal policy studies are undertaken.

7. Conclusion

The World Bank has argued in two recent policy papers that fiscal policy design should seek to ensure macroeconomic stability as well as promote growth and the long-run welfare of a country. The growth impact of the composition of public expenditure is an important aspect of the design of fiscal policy that is consequently the focus of research interest. In this paper we suggest a general model of fiscal policy and growth in an economy with a government that taxes optimally and undertakes public expenditure on (a) education and health facilities, (b) public infrastructure such as roads and bridges, (c) public administration, (d) transfers and public consumption facilities, and (e) debt service. We use this model to explore the impact of shifts in the composition of expenditure on long-run per capita income and other macroeconomic variables. We also set up the model in a way that allows us to study whether fiscal policy is sustainable. This model is solved, the impact of foreign aid, the allocation of human and public capital and fiscal expenditure on per capita income and other macro variables explored.

Foreign aid per capita and the productivity factor both have a positive and linear effect on per capita GDP and welfare. Such a result is clearly what would have been predicted by the theory as inflows of foreign aid are assumed to be used for investment in roads, schools, hospitals, or any other infrastructure that plays an important role in raising productivity. Foreign aid has either no effect, or a slightly negative effect, on the stock of resources absorbed by the public sector. Such a result could be due to some "crowding-in" effect, reflecting the fact that as foreign aid flows in, more investment and production opportunities open up and resources are used more privately and less publicly.

Second, the model suggests that the choice of allocating human capital to market production or to human capital production poses an important trade-off. As the share of human capital that is devoted to human capital production rises (starting from a very low share) income, as well as welfare, initially rises but then falls. In addition, beyond a certain point, the ability to expand the stock of human capital itself is actually diminished. Thus, there exists a growth-maximizing allocation of human capital between final production and the creation of additional human capital. Overall, our model suggests that the larger fraction of human capital should be used for market production[39].

The calibration exercise was also undertaken for different compositions of public investment expenditure for the three country groups. The results of this exercise show that the composition of public investment expenditure matters, as the gains of moving to the optimal allocation between public infrastructure, and education and health facilities are significant. Based on the model and the calibration exercise, a practical

[39] *Of course, this depends on the assumption in our parameter constellations, in particular on the elasticity parameters in the creation of human capital, which add up to 0.6 , a number far below 1.*

rule of thumb seems to be that about two-thirds of public investment should be directed towards public infrastructure that facilitates market production. The remaining third should be split more or less evenly between public investments in facilities that support the provision of health and education. Such a division of resources would maximize (per capita) income and welfare. This is due to the fact that the facilitation of market good production directly increases the availability of public resources, while the other two expenditure categories first have to permeate the economic system before affecting the availability of public resources and thus growth and utility. This result has relevance for policy debate on the composition of expenditure necessary to achieve the Millenium Development Goals (MDGs), which include an income target (halve the number of people below the poverty line) as well as specific targets related to the achievement of health status and access to educational services. The stylized model in this paper suggests that greater emphasis on the health and educational targets relative to investments that may contribute to expansion of market production may result in slower progress on reducing poverty.

During this numerical exercise, attention was also paid to the evolution of public debt. Quite surprisingly, the debt-to-income ratio was almost invariant to all possible investment expenditure compositions. In the considered case the ratio stabilized below 70%. Here the rule of thumb seems to emerge that in the long run debt sustainability does not pose a problem as long as resources for public investments are used in a growth-maximizing way.

We conclude with some observations regarding possible extensions of the model described in this paper. One possibility (described in the appendix) is to estimate the model using time series methods. However, this will only be possible for countries where high-quality time series data are available. An important extension of the model would be to develop the revenue side of fiscal policy in order to study the effects of various forms of taxation on per capita income. Another extension would be to consider the growth implications of variations in the balance between recurrent and capital expenditure by allowing government employment of teachers and doctors. Finally, the debt sustainability tests proposed in this paper could be applied to country-specific studies.

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Appendix

The appendix begins with the out-of-steady-state solution for a simplified version of the model (A.1) and a suggestion of how to estimate the simpler model using time series analysis (A.2).

Appendix A.3 explains how the data used in the calibration was constructed and appendix A.4 lists the countries used. Appendix A.5 lists the data used to compute the graphs shown in section 4 and additional data on the two other country groups which have not been discussed in the paper.

Appendix A.6 does the same with graphs on the long run effects of public expenditure composition that have been omitted in the main part of the paper.

A.1 Out-of-Steady-State Solutions

Since economies are rarely at their steady states, it is, for practical purposes, of great importance to explore what decisions should be taken out of the steady state. There is, however, no analytical solution for our decision variables out of the steady state. Yet, once the parameters have been set and the stationary state of the variables $\{c, e, k, h, g, \lambda_1, \lambda_2, \lambda_3\}\, p$ have been computed, the dynamic programming algorithm developed by Gruene and Semmler (2004) can be used to study the out-of-steady-state dynamics not only of consumption, c, and the use of resources by the public, e_p, but also the state variables: physical capital stock, human capital and public capital stock[40]. Using our parameter set above, next we will compute the out-of-steady-state solutions using a numerical algorithm. This way we can study the following: When, for example, the capital stock of a country is $k < k^*$ (and $h < h^*$, $g < g^*$) the dynamic programming solution allows one to judge whether c should be high or low, how e_p behaves, and how the control variables relate to each other out of the steady state.

[40] *The out of steady state dynamics are analyzed for a simplified version of the model which omits public debt.*

Using the algorithm of Gruene and Semmler (2004) we have solved the model (1)-(4), with u_1 fixed, and thus solely with c and e_p as control variables and k, h and g as state variables[41]. Yet, since the value function (1) is a function of the decision variables c and e_p and the state variables k, h and g (the former again depending on k, h and g) one can only obtain proper graphs by appropriate projections and study the value function and behavior of decision variables in a two-dimensional subspace.

We have solved the model (1)-(4) in the vicinity of the steady state k^*= 65.47, h^* =18.16, g^*=11.47 which was obtained by solving the equation system arising from the Hamiltonian in section 3.1[42]. Next, in the vicinity of the positive steady state we have taken a cube [60,70]×[16,20]×[9,13] for k, u and g, with a subspace for the choice variables [7,8]×[7,8], for c and e_p.

Figure 13 shows four sample trajectories in a three dimensional space, with initial conditions for trajectory 1 (T1): (66.2, 18.9, 11.8) which are above the steady state k^*, h^*, g^*. Trajectory 2 (T2) has initial conditions: (67.5, 18.7, 11.4) and is, thus, far to the right of the steady state k^*, h^*, g^*. Trajectory 3 (T3): (63.2, 16.8, 11.1) starts below the steady state k^*, h^*, g^*. Finally, Trajectory 4 (T4) starts with k= 68.2 above, with h= 16.5 below and with g at the steady state.

As one can observe in Figure 13, the correctly taken optimal decisions c and e_p make the initial states of state variables converging toward the steady state k^*, h^*, g^* for all four sample trajectories. Note that trajectories (T1), (T2), (T3) and (T4) do not only exhibit some irregular features but also cross each other. This comes from the fact that we follow the trajectories in a 2-dim subspace state variables only, namely in the k–h space. For graphical purposes, the third state variable g is fixed in Figure 13.

If one stores the decision variables c and e_p at each grid point of the 3-dim cube of k, h and g one can then plot them in a 3-dim space where the height represents the numerical value of the decision variable. In Figure 14, for example, the height stands for the value of c, and the other two axes represent k and h.

In Figure 14 it is clearly visible that if $k>k^*$ and $h>h^*$ the optimal consumption c is required to be high, above its steady state. The reverse holds for the space $k<k^*$ and $h<h^*$. Here c has to be below its steady state. This behavior of the optimal c is a result that one would also expect from economic intuition.

[41] Note that this simplified version omits the debt equation (5) of the model.
[42] There is, however, a second steady state which was neglected because of negative values of some state variables.

Figure 13. Out of Steady State Dynamics

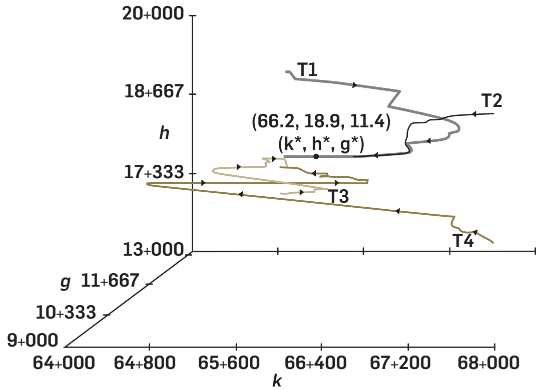
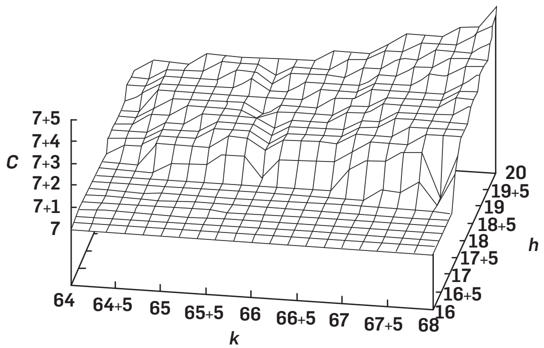


Figure 14. Decision Variable c in the k–h space



On the other hand as Figure 15 shows the decision variable e_p is not so monotonically dependent on k and h. In some regions e_p is higher than in others. Note that we have neglected here to study the behavior of c depending on government capital, g. This also can be easily done, but this might not be so relevant in the context of our study. Next, we want to study the dependence of the total use of public resources, e_p, on private capital, k, and public capital, g.

Figure 16 shows how the decision variable e_p behaves in the k–g space. We can observe that e_p will be high above the steady state, for $g<g^*$, and low below the steady state, for $g < g^*$. The decision variable e_p does not seem to depend much on the level of the capital stock, k. Finally, we want to make a comment on the constraints that we have put on the control variables, c and e_p. For both the lower constraint is 7 and the upper is 8 so that all optimal c and e_p are staying within those constraints. But note that with $c^*= 7.04$ and*=7.53 e_p the decision variables are above the lower constraints of 7. Putting such constraints on the decision variables just means that the decision variables cannot go fast enough down and thus the state variables that are affected by this are not going up fast enough. This is the reason why the trajectory (T4) in Figure 13

Figure 15. **Decision Variable e_p in the k–h space**

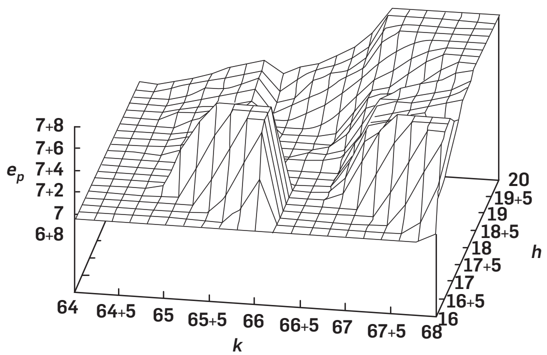
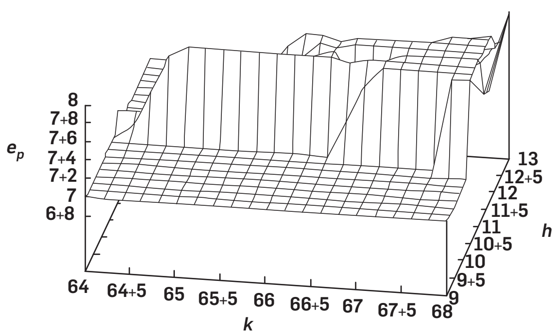


Figure 16. **Decision Variable e_p in the k–g space**



first moves far down (to the left) in the state variable k and then rises again moving toward its steady-state value. The above exercise with constraints for the decision variables is nevertheless of great importance for practical economic policy. It basically means that the decision variables do not have to be exactly at their optimal values in order to exhibit convergence dynamics. For actual economies an exact optimal control is indeed hard to achieve, since for actual economies, there is model uncertainty – which model fits the economy – as well as data uncertainty (see Brock et al., 2003). Our computations with constraints for the decision variables then means that the decision variables need to be only above or below their steady-state values in the appropriate state space in order to fulfill the requirement to be roughly optimal. This aspect appears to us of great practical importance, since decisions for c and e_p have to be only approximately correct.

A.2 The Estimable Form of the Model

Next we want to spell out some implications of how to estimate our model with time series methods. Even for our simplified social planner model of eqns. (1) to (4) it could be too ambitious to estimate the model employing Euler equations, derived from the first order conditions for the two control variables for consumption, c, and total use of resources, e_p, and the decision variable u_1. As the model is written, it gives us the outcome of the social optimum. We propose to reduce the model further, and treat, for the purpose of a time series estimation, only consumption as decision variable. The other choice variables, e_p and u_1 are just treated as historical variables, as time series observations. This is also likely to inject some realistic features into the model. For a time series study we thus propose to include only one Euler equation in the time series estimation procedure. We suggest to only use the equations (2)-(4) and the equation for $\dot{c}$, which can be derived from the first order condition of the decision variable c as

$$\frac{\dot{c}}{c} = -\frac{\dot{\lambda}_1}{\lambda_1} \rightarrow \dot{c} = c\left(A\alpha k^{\alpha-1}\ (u_1 h)^{\beta}\ (v_1 g)^{\gamma} - \rho - \delta_k\right) \tag{29}$$

The model reflects now the fact that it is only optimized with respect to private consumption. The decision making process for all the public variables might be considered too complex to be presumed as the result of some optimization process. Yet, in the normative sense, as we discuss below, we still might consider the other public decision variables as choice variables so as to give us a guidance to welfare improving policies. Above, we have undertaken some calibration and comparative static study in order to explore the impact of fiscal policy decisions on per capita income and welfare.

A.3 Data

Two different sources were used for gathering the data necessary for the calibration and comparative static exercises. First, information on GDP, population size and foreign aid flows was taken from the World Bank's World Development Indicators (WDI). Second, information on the public expenditure composition was taken from the Government Finance Statistics (GFS). In order to make the data compatible with the model, only category 7 (and its subcategories) of the GFS were used. Public infrastructure investment, $v_1\alpha_1 e_p$, consists of "Economic affairs" (704), "Environmental protection" (705) and "Housing and community amenities" (706). Public expenditure on health, $v_2\alpha_1 e_p$, was taken from "Health" (707). Public expenditure on education, $v_3\alpha_1 e_p$, consists of "Education" (709). "Recreation, culture, and religion" (708) and "Social protection" (710) was subsumed under public transfers, $\alpha_2 e_p$. Public consumption and debt payments, $\alpha_3 e_p$ and $\alpha_4 e_p$, were defined as the sum of "General public services" (701), "Defense" (702) and "Public order and safety" (703). Since the information on debt service is very spotty, we chose $\alpha_4 = 0.10$ in a rough approximation and set the accordingly decreased above sum equal to α_3.

We follow the IMF Development Committee (2006) in its classification of countries in low, lower-middle and upper-middle income countries. However, due to the bad quality of the data in the GFS, we dropped some countries and reduced the list of countries to the ones listed in appendix A.4.

A.4 Country Groups

Table 3. Reduced List of Countries

Low-Income	Lower-Middle Income	Upper-Middle Income
Bhutan	Belarus	Argentina
Burundi	Bolivia	Croatia
Ethiopia	Bulgaria	Czech Republic
India	Dominican Republic	Estonia
Moldova	Egypt	Hungary
Myanmar	El Salvador	Lithuania
Pakistan	Iran	Mauritius
Yemen	Jamaica	Mongolia
Zambia	Maldives	Panama
	Morocco	Poland
	Philippines	Uruguay
	Romania	
	Sri Lanka	
	Thailand	
	Tunisia	

A.5 Appendix to Section 4

Table 4. Effect of Foreign Aid, i_p^f, on the Steady-State Variables (Low-Income Group)

i_p^f	e_p	c	k	h	g	b	y	U
2.0	1539.3	890.4	10848	1152.6	9005	2531.8	3451.6	89.70
2.2	1539.1	891.0	10850	1152.8	9006	2531.5	3452.2	89.71
2.4	1538.9	891.6	10851	1152.9	9008	2531.2	3452.7	89.72
2.6	1538.8	892.1	10853	1153.0	9010	2530.8	3453.2	89.73
2.8	1538.6	892.7	10855	1153.2	9012	2530.5	3453.8	89.73
3.0	1538.4	893.2	10856	1153.3	9014	2530.2	3454.3	89.74
3.2	1538.2	893.8	10858	1153.4	9015	2529.9	3454.8	89.75
3.4	1538.0	894.4	10860	1153.6	9017	2529.6	3455.4	89.76
3.6	1537.8	894.9	10861	1153.7	9019	2529.3	3455.9	89.80
3.8	1537.6	895.5	10863	1153.8	9021	2529.0	3456.4	89.77
4.0	1537.5	896.0	10865	1153.9	9023	2528.7	3457.0	89.78

Table 5. Effect of the Productivity Factor, A, on the Steady-State Variables (Low-Income Group)

A	e_p	c	k	h	g	b	y	U
1.000	1538	894	10859	1153	9016	2530	3455	89.8
1.005	1589	923	11214	1175	9311	2613	3568	90.2
1.010	1641	953	11580	1196	9614	2699	3684	90.6
1.015	1694	984	11955	1218	9926	2786	3804	91.1
1.020	1749	1015	12341	1240	10246	2877	3927	91.5
1.025	1805	1048	12737	1262	10575	2969	4053	91.9
1.030	1863	1081	13144	1284	10913	3065	4182	92.3
1.035	1923	1115	13562	1307	11259	3163	4315	92.8
1.040	1984	1150	13992	1330	11615	3263	4452	93.2
1.045	2047	1186	14432	1354	11981	3366	4592	93.6
1.050	2111	1223	14885	1377	12356	3472	4736	94.0
1.055	2177	1261	15349	1401	12742	3581	4884	94.4
1.060	2245	1300	15825	1425	13137	3692	5035	94.9

Table 6. Effect of the Fraction of Human Capital Used in Market Goods Production, u_1, on the Steady-State Variables (Low-Income Group)

u_1	e_p	c	k	h	g	b	y	U
0.05	9	15	104	144	98	14	33	32.4
0.10	39	33	321	253	274	64	102	44.4
0.15	93	64	700	383	588	152	223	53.7
0.20	168	107	1230	517	1028	276	391	60.8
0.25	263	162	1899	648	1582	433	604	66.4
0.30	376	227	2691	774	2239	618	856	71.0
0.35	503	300	3586	893	2982	828	1141	74.9
0.40	642	380	4563	1002	3793	1056	1452	78.1
0.45	789	464	5597	1100	4650	1298	1781	80.8
0.50	940	551	6657	1184	5530	1547	2118	83.1
0.55	1090	637	7710	1252	6404	1793	2453	85.1
0.60	1233	719	8716	1303	7239	2028	2773	86.8
0.65	1363	793	9628	1332	7995	2242	3063	88.1
0.70	1471	855	10386	1338	8624	2419	3305	89.2
0.75	1547	899	10918	1315	9065	2544	3474	89.8
0.80	1576	916	11123	1256	9236	2592	3539	90.1
0.85	1538	894	10859	1153	9016	2530	3455	89.8
0.90	1399	814	9882	989	8206	2301	3144	88.5
0.95	1084	633	7665	721	6366	1782	2439	85.0

Table 7. Effects of Foreign Aid, i_p^f, on Steady-State Variables (Lower-Middle-Income Group)

i_p^f	e_p	c	k	h	g	b	y	U
3.00	5824.8	4659.7	45470	2742.7	42050	8642.2	14467.8	104.38
3.20	5824.7	4660.3	45472	2742.8	42052	8642.0	14468.4	104.38
3.40	5824.6	4660.9	45474	2742.9	42055	8641.8	14469.0	104.38
3.60	5824.5	4661.4	45476	2743.0	42057	8641.6	14469.6	104.38
3.80	5824.3	4662.0	45478	2743.0	42059	8641.4	14470.2	104.38
4.00	5824.2	4662.6	45480	2743.1	42061	8641.3	14470.8	104.38
4.20	5824.1	4663.1	45481	2743.2	42064	8641.1	14471.4	104.38
4.40	5823.9	4663.7	45483	2743.3	42066	8640.9	14472.0	104.38
4.60	5823.8	4664.3	45485	2743.4	42068	8640.7	14472.6	104.40
4.80	5823.7	4664.8	45487	2743.5	42071	8640.5	14473.2	104.39
5.00	5823.6	4665.4	45489	2743.5	42073	8640.3	14473.8	104.39

Table 8. Effects of the Productivity Factor, A, on the Steady-State Variables (Lower-Middle-Income Group)

A	e_p	c	k	h	g	b	y	U
1.180	5824	4663	45481	2743	42063	8641	14471	104.4
1.185	5986	4792	46742	2786	43229	8881	14873	104.7
1.190	6152	4924	48034	2829	44423	9127	15283	105.1
1.195	6321	5059	49355	2872	45645	9378	15704	105.4
1.200	6494	5198	50707	2916	46895	9636	16134	105.7
1.205	6672	5339	52091	2961	48174	9899	16574	106.1
1.210	6853	5484	53506	3006	49483	10168	17025	106.4
1.215	7039	5632	54953	3051	50821	10443	17485	106.8
1.220	7229	5784	56434	3097	52190	10725	17956	107.1
1.225	7423	5939	57948	3144	53590	11013	18438	107.4
1.230	7621	6097	59496	3191	55021	11308	18931	107.8
1.235	7824	6259	61079	3238	56485	11609	19434	108.1
1.240	8032	6425	62698	3286	57982	11917	19949	108.4

Table 9. Effect of u_1, the Fraction of Human Capital Used in Market Goods Production, on the Steady-State Variables (Lower-Middle-Income Group)

u_1	e_p	c	k	h	g	b	y	U
0.05	35	41	328	278	316	52	104	44.3
0.10	154	137	1260	574	1176	229	401	59.8
0.15	359	300	2856	894	2651	532	909	69.8
0.20	644	528	5082	1217	4710	956	1617	76.9
0.25	1004	815	7889	1532	7305	1490	2510	82.4
0.30	1430	1155	11211	1835	10376	2122	3567	86.8
0.35	1912	1540	14968	2119	13850	2836	4763	90.4
0.40	2437	1959	19068	2380	17641	3616	6067	93.5
0.45	2993	2403	23404	2613	21651	4441	7447	96.0
0.50	3564	2859	27853	2813	25764	5288	8862	98.2
0.55	4130	3311	32272	2977	29849	6128	10268	100.1
0.60	4672	3743	36492	3097	33752	6931	11611	101.6
0.65	5162	4134	40316	3168	37287	7659	12828	102.9
0.70	5570	4460	43498	3181	40229	8264	13840	103.8
0.75	5856	4688	45728	3126	42292	8688	14550	104.5
0.80	5967	4777	46591	2988	43089	8852	14824	104.7
0.85	5824	4663	45481	2743	42063	8641	14471	104.4
0.90	5299	4243	41383	2351	38274	7862	13167	103.2
0.95	4106	3291	32082	1713	29674	6092	10208	100.0

Table 10. Effects of Foreign Aid, i_p^f, on Steady-State Variables (Upper-Middle-Income Group)

i_p^f	e_p	c	k	h	g	b	y	U
3.0	10068.2	11326.4	89637	3597.5	68113	13335.4	28520.8	106.26
3.2	10068.1	11327.3	89640	3597.6	68116	13335.3	28521.8	106.26
3.4	10068.1	11328.1	89643	3597.7	68119	13335.2	28522.8	106.27
3.6	10068.0	11328.9	89646	3597.8	68122	13335.1	28523.8	106.27
3.8	10067.9	11329.8	89649	3597.9	68125	13334.9	28524.8	106.27
4.0	10067.8	11330.6	89652	3598.0	68128	13334.8	28525.8	106.27
4.2	10067.7	11331.4	89656	3598.0	68131	13334.7	28526.8	106.27
4.4	10067.6	11332.3	89659	3598.1	68134	13334.6	28527.7	106.27
4.6	10067.5	11333.1	89662	3598.2	68137	13334.5	28528.7	106.30
4.8	10067.4	11333.9	89665	3598.3	68140	13334.4	28529.7	106.27
5.0	10067.4	11334.8	89668	3598.4	68143	13334.3	28530.7	106.27

Table 11. Effects of the Productivity Factor, A, on the Steady-State Variables (Upper-Middle-Income Group)

A	e_p	c	k	h	g	b	y	U
1.450	10068	11331	89654	3598	68130	13335	28526	106.3
1.455	10295	11586	91674	3643	69664	13636	29169	106.5
1.460	10526	11846	93733	3689	71228	13942	29824	106.8
1.465	10762	12111	95830	3735	72822	14254	30491	107.0
1.470	11002	12381	97967	3782	74446	14572	31171	107.3
1.475	11247	12656	100145	3829	76100	14897	31864	107.5
1.480	11496	12936	102363	3876	77785	15227	32570	107.8
1.485	11750	13221	104622	3924	79502	15563	33289	108.0
1.490	12009	13512	106924	3973	81251	15906	34021	108.3
1.495	12272	13808	109268	4021	83032	16255	34767	108.5
1.500	12541	14110	111656	4071	84846	16610	35527	108.8
1.505	12814	14417	114088	4120	86693	16972	36301	109.0
1.510	13092	14729	116564	4170	88574	17341	37089	109.3

Table 12. Effect of u_1, the Fraction of Human Capital Used in Market Goods Production, on the Steady-State Variables (Upper-Middle-Income Group)

u_1	e_p	c	k	h	g	b	y	U
0.05	62	87	623	355	488	82	198	49.9
0.10	269	319	2465	749	1886	356	784	65.1
0.15	623	717	5610	1170	4276	825	1785	74.4
0.20	1116	1271	10000	1594	7611	1478	3182	81.1
0.25	1738	1970	15534	2008	11815	2302	4943	86.1
0.30	2474	2797	22083	2405	16791	3277	7027	90.1
0.35	3307	3732	29491	2778	22419	4380	9383	93.5
0.40	4215	4753	37575	3120	28561	5583	11956	96.3
0.45	5176	5833	46124	3426	35056	6856	14676	98.6
0.50	6162	6940	54894	3690	41719	8161	17466	100.6
0.55	7141	8041	63606	3904	48338	9458	20238	102.3
0.60	8076	9091	71927	4062	54660	10697	22886	103.7
0.65	8923	10043	79465	4155	60387	11819	25284	104.9
0.70	9629	10836	85739	4173	65153	12753	27280	105.8
0.75	10123	11391	90136	4100	68494	13408	28680	106.3
0.80	10314	11606	91837	3919	69787	13661	29221	106.5
0.85	10068	11329	89648	3598	68123	13335	28524	106.3
0.90	9160	10309	81569	3083	61985	12132	25954	105.2
0.95	7099	7993	63232	2246	48054	9402	20119	102.2

A.6 Appendix to Section 5

Since the calibration exercise for the composition of public expenditure involves too many parameters to list all data in tables, the depiction of the lower-middle- and upper-middle-income cases is limited to graphs. As one can see, these graphs show the same qualitative effects as the low-income group's that were discussed in the main part of the paper.

Figure 17. Effects of Expenditure Composition Changes on the per capita Parameters in the Lower-Middle-Income Case

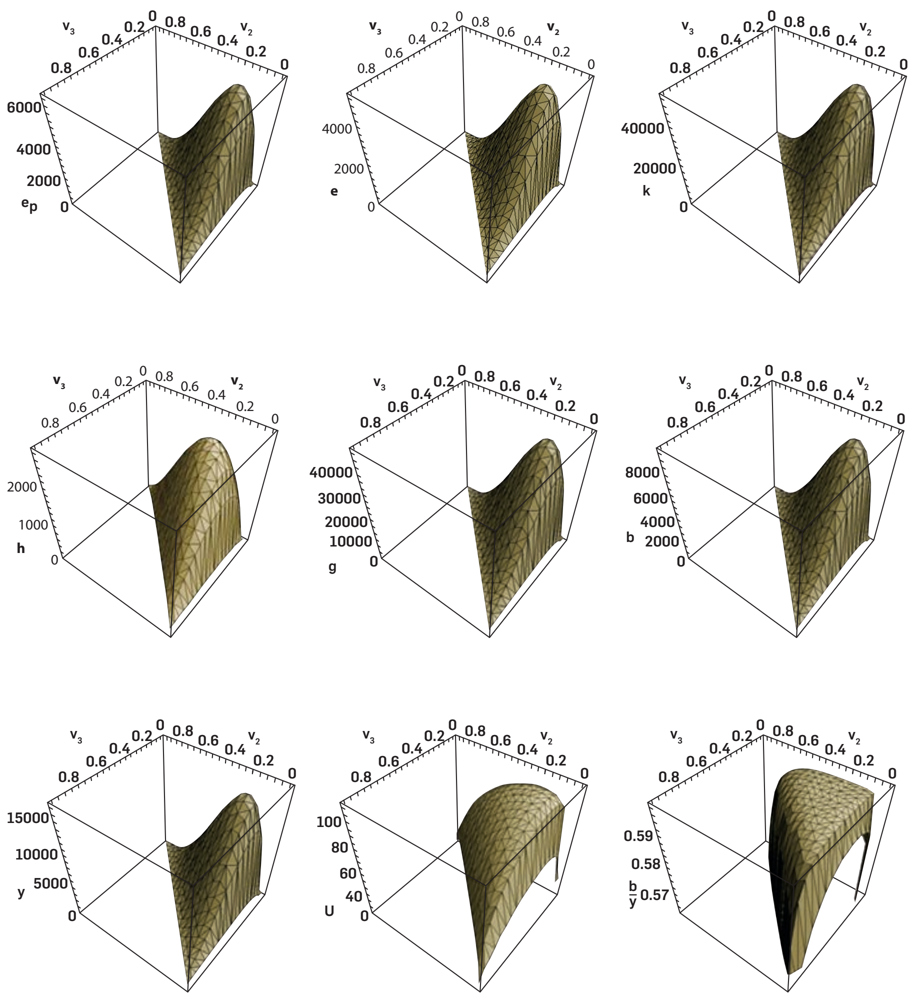


Figure 18. Effects of Expenditure Composition Changes on the per capita Parameters in the Upper-Middle-Income Case